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Chemistry. — *"The Metastability of the Elements and Chemical Compounds in consequence of Enantiotropy or Monotropy, VIII.*
By Prof. ERNST COHEN and Dr. W. D. HELDERMAN.

(Communicated at the meeting of June 28, 1924.)

1. In our fourth paper on the subject under the above title¹⁾ it was shown, that BRIDGMAN's determination of the change of volume which takes place when ammonium nitrate passes from IV into III (at 32°.3 C.) has an error of about 9⁰/₀, and this in consequence of the persistent retardations which can occur during this transformation. This was quantitatively another proof that the physical constants of solid substances which we know up to the present moment generally relate to metastable mixtures of unknown composition of the modifications of those substances. At the same time it was again insisted upon that no confidence can be put in those constants, unless they are determined for the pure modifications.

It stands to reason (in the paper above mentioned it was indeed pointed out) that values, calculated with those constants that have been thus erroneously determined, are also wrong, unless accidental compensation of experimental errors should have played a part in the calculation.

Now BRIDGMAN²⁾ in his investigation mentioned above has found: 4.66 $\frac{\text{gr. calories}}{\text{gm.}}$ for the transition heat III \rightarrow IV at the transition point, by means of data he experimentally determined:

$$\left(\frac{dT}{dp} = 0.0311 \frac{\text{degree}}{\text{kg.cm.}}; (v_{III} - v_{IV}) = 0.02026 \frac{\text{ccm.}}{\text{gr.}} \right)$$

according to the equation of CLAPEYRON—CLAUSIUS:

$$\frac{dT}{dp} = \frac{T(v_{III} - v_{IV})}{W}.$$

The question now arises: is this figure correct, or has it, just like the value $(v_{III} - v_{IV})$ determined by BRIDGMAN, as great an error (about 9⁰/₀)? From the way he made his $\frac{dT}{dp}$ determinations³⁾ it is obvious that they

¹⁾ These Proceedings, 27, 65 (1924); Zeitschr. f. physik. Chemie 109, 81 (1924).

²⁾ Proc. Americ. Acad. of Arts and Sciences 51, 581 (1916).

³⁾ Proc. Americ. Acad. of Arts and Sciences 51, 55 (1915); Phys. Rev. 3, 126 (1914); 6, 1 (1915).

were fixed with sufficient accuracy. Consequently it was necessary that the transition heat at the transition temperature should be most accurately determined.

2. It is true, that BELLATI and ROMANESE ¹⁾ have found for it the value of $5.02 \frac{\text{gm. cal.}}{\text{gm.}}$, but it is not sufficiently certain that they made their investigations with the pure modification III and IV, while moreover many objections may be raised against the method they followed. Their manner of procedure was as follows:

First of all they determined the transition point by the thermometrical method. By heating this was found to be 35° , by cooling 31° . Quite arbitrarily they fixed the transition temperature at 31° . Then the mean specific heat of modification IV between 0 and 26° was determined by putting weighed quantities of it in a calorimeter, filled with turpentine, and determining the consequent accompanying heat effect. The same procedure was then carried out with modification III in a temperature interval from $82-31^\circ$, cooling it from a higher temperature than the transition point to one below this temperature. The results thus obtained are found in table 1.

TABLE I.

Nº.	T	t	Q
1	$0^\circ.05$	$24^\circ.79$	10.00
2	0.17	25.79	10.28
3	0.18	26.13	10.76
4	39.99	27.10	9.95
5	39.84	25.74	10.13
6	78.3	27.55	23.04
7	80.9	27.00	24.58

Here T and t represent the initial, respectively the final temperature of the salt, and Q the amount of heat given off, when one gram of the salt was cooled from the higher temperature down to the lower.

3. *Diagrammatically* BELLATI and ROMANESE have found from these data:

mean specific heat between 0 and 31° of modification IV : 0.407

" " " " 31 " $82^\circ.5$ " " II : 0.355

Heat of transition III \rightarrow IV at 31° : 5.02 gm. cal. per gram NH_4NO_3 .

¹⁾ Atti del R. Istituto Veneto (6) 4, 1395 (1886); also Il nuovo Cimento (3) 21. 1 (1887).

By computation we find from the above table:

From experiment 1 :	mean specific heat IV :	0.404	} mean 0.407
" " 2 :	" " " IV :	0.401	
" " 3 :	" " " IV :	0.415	

The divergencies between the different experiments therefore are not inconsiderable.

Further the computation gives:

Mean specific heat III : 0.356.

Heat of transition III \rightarrow IV 4.99 gram calories per gram NH_4NO_3 .

The objections against calculating the heat of transition at the transition temperature by means of the mean specific heats of the two modifications, determined for great temperature intervals above and below the transition temperature instead of by means of the true spec. heat of those modifications at the temperature of transition, are obvious.

4. Some time ago, when our investigations had already begun, MONDAIN MONVAL¹⁾ gave a second, more incidental determination of the heat of transition when he calculated from his investigations this heat effect as the difference of the fictitious heats of solution of the modifications III and IV of ammonium nitrate at the transition temperature. His determinations however are such (we hope to return to this subject in another connection later on), that a priori not much value can be attached to his results.

The Method employed.

5. When, of the two pure modifications III and IV at the transition temperature (which according to our previous investigations lies at $32^\circ.3$ at 1 atm. pressure), we determine the heat of solution in some medium, taking care that the final state is the same for equal quantities of the two modifications, the difference of these heats of solution gives us the desired heat of transition at the temperature of the experiment.

In connection with another investigation which will be described later on, we have experimentally fixed the curves which at the transition temperature represent the intermediate heats of solution of the modifications III and IV as function of the concentrations of aqueous solutions of ammonium nitrate. If these ordinates represent these intermediate heats of solution (per mole), the abscissae the concentrations of the solutions used, the desired heat of transition (per mole) is equal to the difference of the ordinates of any pair of points in the two curves which have the same abscissa.

¹⁾ C. R. Paris 177, 175 (1923).

The Materials used.

6. An already very pure commercial preparation was recrystallized a few times from distilled water. Then impurities could not be found in 10 gm¹). A solution of the last crystallization, saturated at 100°, was filtered through a hot water funnel and allowed to stand. From the salt crystallized out we prepared some kilograms of each of the two pure modifications III and IV.

a. Modification III.

(Region of Stability 32°.3 — 84°.2 C.).

7. As will appear from what follows special precautions are absolutely necessary for the preparation of the different modifications and the control whether the pure modification, one expects to have, is present. If these precautions are not taken the investigator exposes himself to many complications during the calorimetric measurements. Modification III has been prepared in two different ways. (Preparation III A and III B.)

8. (Preparation III A). Part of the salt, mentioned in par. 6, which had crystallized from a saturated solution at 100°, between this temperature and 18°, was placed in moist condition in a drying oven, in which it was kept at 75° for two months. From time to time it was powdered very fine in a mortar in order to renew the surface and to open vacuoles which eventually might be present. Finally we transferred it to an desiccator, in which it was kept at about 40° for a few weeks over P_2O_5 . From this stock we continually used a part for the caloric experiments.

9. In order to make sure that the salt, thus prepared, was really the pure modification III, we determined the density at 32°.3 (the transition temperature). ERNST COHEN and J. KOOY²) had found for it $d_{40}^{32.3} = 1.654$, whereas the density of modification IV is $d_{40}^{32.3} = 1.716$. We used xylene in the pycnometer as liquid which previously had been distilled in presence of P_2O_5 .

In two independent determinations with two different pycnometers, we found for our preparation $d_{40}^{32.3} = 1.701$ and 1.701, the next day, with a new quantity of salt 1.700. For all the determinations we used about 15 gm. of salt. When the salt of the last determination had been for 24 hours in contact with xylene in the pycnometer, we found $d_{40}^{32.3} = 1.705$.

10. Now it was possible that our preparation III A had originally been the pure modification III, but that during the pycnometric determination

¹) E. MERCK, Prüfung chemischer Reagentien auf Reinheit, 2. Aufl. 1912, S. 73.

²) These Proceedings, 27, 65 (1924); Zeitschr. f. physik. Chemie 109, 82 (1924).

a partial transformation into IV had taken place. According to former investigations with cadmium iodide¹⁾ it did not seem probable that the dry salt in the pycnometer during weighing (at room temperature) had stabilized — partially or totally — to IV, but it was possible that III had partially been transformed into IV, after xylene at room temperature had been added in the pycnometer, and the apparatus had been heated in a thermostat to 32°.3, and had been evacuated. That this was indeed the case was proved by the experiments which we made with a new preparation of modification III (III B).

11. The preparation of III B was as follows:

A large quantity of recrystallized NH_4NO_3 was totally dissolved and placed in a crystallizing dish in a drying oven which was kept at 60°. After 4 days and nights the salt was dry; it was powdered and kept over P_2O_5 in a desiccator at 60°. After 4 days and nights we determined the density in the following way:

When the dry salt had been weighed in the pycnometer, we placed the latter in the thermostat at 32°.3. When the salt had assumed this temperature, we filled up the pycnometer with the xylene, mentioned above, which had previously likewise been heated to 32°.3. We found now: $d_{40}^{32.3} = 1.651$ and 1.653.

So the preparation was indeed the pure modification III, and is metastable at room temperature.

Our large stock of this preparation which was used later on in the caloric experiments, was always kept in a desiccator over P_2O_5 at 60°.

12. In order to prove that the manipulations for the determinations of the density of III A really might have given rise to partial stabilization, the density at 32°.3 of two quantities of III B was investigated, just as had been done with III A, that is to say: after weighing the dry salt in the pycnometers at room temperature, xylene, not previously heated, was added to them. Then the pycnometers were placed in the thermostat at 32°.3. Now we found $d_{40}^{32.3} = 1.693$ and 1.698. Partial stabilization had indeed taken place.

b. Modification IV.

(Region of Stability — 16° up to 32°.3 C.).

13. This preparation was also obtained in two different ways (IV A and IV B), IV A in the following manner:

The moist salt, crystallized from a solution saturated at 100° between that temperature and 18° was placed under a desiccator jar by the side

¹⁾ ERNST COHEN and A. L. TH. MOESVELD, Verslag afd. Natuurkunde Kon. Akademie v. Wet. 28, 581, 602. (1920) Zeitschr. f. physik. Chemie 94, 471, 482 (1920).

of a dish filled with water. When it had remained for a month at 18° , we substituted sulphuric acid for the water, and removed the air from the jar. Every day the salt was stirred. After two weeks we replaced the sulphuric acid by P_2O_5 , and then dried for six weeks more. During this period the salt was powdered in a mortar every two days. At last we determined the density at 32.3° , observing the precautions mentioned with preparation IIIB (par. 11).

We found: $d_{40}^{32.3} = 1.713$ and 1.714 , whereas, according to the determinations of ERNST COHEN and J. KOOY¹⁾, at that temperature the density is 1.716 . It may be remarked that more confidence is to be placed in the latter figure than in the one found with the pycnometer, the accuracy of which is here only about 3 units of the third decimal place.

So we infer that our preparation IVA is a pure modification IV. It was kept over P_2O_5 in a desiccator at room temperature. In the caloric experiments we continually used part of it. In the last experiments (see Table II) with the solutions of 68.75 per cent we experimented with a freshly prepared preparation of IV, the density of which was $d_{40}^{32.30} = 1.716$.

14. During a second preparation of modification IV (preparation IVB), we had experiences which may serve as a warning. We started with a quantity of NH_4NO_3 which for some time had been heated above the transition point. In order to stabilize the whole mass to IV, we moistened it with water, and let it stand at 18° . After a few days we dried the salt in vacuo over sulphuric acid, then over P_2O_5 . In two independent determinations we found $d_{40}^{32.3} = 1.647$.

As from our investigations on cadmium iodide we knew²⁾ that this salt easily stabilizes when it is in contact with toluene, distilled in presence of P_2O_5 , we tried to accelerate the transformation of NH_4NO_3 into IV under toluene at 18° . This did not take place even after some days. Then we removed the toluene by washing with alcohol and ether, and put the salt under an aqueous, saturated solution of NH_4NO_3 , with which it was left in contact at 18° for 24 hours. After drying for some days in vacuo, over sulphuric acid, and afterwards over P_2O_5 , the density (originally 1.647) was again determined. It had remained unchanged.

When the salt had been once more under the aqueous solution for two days, the density had increased to 1.677 (1.677); after having been under toluene for 24 hours, the salt showed a density of 1.680 (1.678), three days and nights later 1.680 (1.678).

Recapitulating we see that complete stabilization of modification III into IV at 18° did not take place, neither when it is in contact with a saturated aqueous solution, nor with the toluene or xylene preparations used.

¹⁾ These Proceedings **27**, 65 (1924); Zeitschr. f. physik. Chemie **109**, 81 (1924).

²⁾ ERNST COHEN and A. L. TH. MOESVELD, Verslag etc. **28**, 581, 602 (1920); Zeitschr. f. physik. Chemie **94**, 471, 482 (1920).

The caloric Measurements.

15. For these measurements we have made use of the electrical, adiabatic calorimeter for the determination of heats of solution which was recently described by ERNST COHEN, A. L. TH. MOESVELD and W. D. HELDERMAN ¹⁾. For the experimental method and the calculation of the determinations we refer to the paper mentioned, reminding our readers that the accuracy of the determinations may be put at about 3 pro mille.

All the determinations made by us hold good for the temperature of the transition point of modification III into IV ($32^{\circ}.3$ C).

16. Before giving the numerical results in table 2, we wish to make a remark about the preparation of the solutions, in which the intermediate heats of solution of the two modifications were determined, as well as about the way in which the flasks, containing the preparation to be solved, were prepared.

As ammonium nitrate is a hygroscopic substance, special precautions must be taken in weighing the salt which is put into the flasks, (up to the time of filling the flasks it had continually been kept over $P_2 O_5$). Our manner of procedure was as follows: in the afternoon the necessary quantities of salt were roughly weighed in the flasks as quickly as possible. Then they were placed again in the desiccator over $P_2 O_5$, and the next day the weighings were done very accurately (flasks closed with stoppers). Then the flasks were sealed.

The flasks which contained modification III (metastable at room temperature) were always kept over $P_2 O_5$ at 60° , after the preliminary weighing had been done. After sealing, up to the moment when they were placed in the calorimeter, they were also kept at that temperature.

17. The stock of the solutions was prepared by weighing salt and water on a large balance, which could bear about 20 Kgm. Duplicate estimations were always made of the concentrations by determining the density of the solution. For this we used the equation we gave some time ago for the relation between concentration and specific volume of ammonium nitrate solutions at $32^{\circ}.3$)²⁾. The solutions which were super-saturated at room temperature were kept in flasks standing in a thermostat which was kept at about 30° . Before transferring them to the calorimeter we warmed the solutions up to about 32° , and by means of the heating-stirrer we then raised the temperature to the temperature of the experiment ($32^{\circ}.3$). All the weighings were reduced to vacuo.

18. Table II gives a survey of the calorimetric determinations.

¹⁾ These Proceedings 27, 656 (1924).

²⁾ ERNST COHEN, W. D. HELDERMAN and A. L. TH. MOESVELD, These Proceedings 27, 565 (1924).

TABLE II.

Intermediate heats of solution of ammonium nitrate modification IV and III in solutions of ammonium nitrate concentration. Temp. 32° 3 C. (transition temperature).

Name of the modification.	Weight of the solution in gm.	Weight of the salt in gm.	Correction for Radiation in degrees Celsius	Initial concentration in gm. per 100 gm. solution	Final concentration in gm. per 100 gm. solution	Mean concentration in gm. per 100 gm. solution.	Intermediate heat of solution in gm. calories per mole.	Intermediate heat of solution in gm. cal. per mole corrected	Mean interm. heat of sol. in gm. calories per mole found.	Interm. heat of sol. in gm. cal. per mole calculated	Interm. heat of sol. in gm. cal. per mole calc.—found.
IV	1125.0	33.4363	−0.001	12.52	15.04	13.78	−4901	−4899			
IV	1120.5	33.3132	+0.000 ⁵	12.55	15.08	13.81	−4909	−4906	−4903	−4902	+ 1
IV	1157.3	33.4923	±0.000	12.52	14.98	13.75	−4904	−4904			
III	1138.7	33.2520	−0.000 ⁵	12.52	14.98	13.75	−4512	−4512			
III	1138.7	33.2558	−0.001	12.52	14.98	13.75	−4505	−4505	−4509	−4502	+ 7
IV	1212.0	32.8705	±0.000	26.53	28.47	27.50	−4123	−4123			
IV	1216.5	32.9919	+0.001	26.53	28.47	27.50	−4116	−4116	−4120	−4125	− 5
III	1171.1	33.0090	+0.001	26.47	28.49	27.48	−3715	−3716			
III	1128.4	32.8477	−0.001	26.46	28.54	27.50	−3718	−3718	−3717	−3725	− 8
IV	1240.9	43.8249	±0.000	40.30	42.34	41.32	−3581	−3579			
IV	1246.1	43.7685	−0.003	40.35	42.39	41.37	−3587	−3583	−3584	−3577	+ 7
IV	1288.1	43.0269	+0.003	40.29	42.22	41.25	−3589	−3589			
III	1313.0	43.6582	−0.001	40.29	42.22	41.25	−3190	−3190			
III	1287.1	43.2247	+0.000 ⁵	40.28	42.22	41.25	−3183	−3183	−3187	3177	+10
IV	1315.7	41.3644	−0.001	54.07	55.47	54.77	−3200	−3200			
IV	1322.1	41.5644	−0.000 ⁵	54.07	55.47	54.77	−3199	−3199	−3200	−3197	+ 3
III	1301.4	40.9142	−0.001	54.07	55.47	54.77	−2799	−2799			
III	1321.0	41.5153	+0.002 ⁵	54.07	55.47	54.77	−2792	−2792	−2796	−2797	− 1
IV	1401.4	31.7401	+0.002	68.40	69.10	68.75	−2935	−2935			
IV	1398.6	31.6687	+0.001 ⁶	68.40	69.10	68.75	−2944	−2944	−2940	−2945	− 5
III	1420.0	32.1607	+0.002	68.40	69.10	68.75	−2540	−2540			
III	1407.1	31.8874	±0.000	68.40	69.10	68.75	−2538	−2538	−2539	−2545	− 6

It is to be noted that the figures of the ninth column were obtained by converting the values of the eighth column to the same mean concentration in so far these concentrations were not equal.

19. As the absolute value of the heats of solution measured depends on the value of the E. M. F. of the WESTON-cell at 25° 0 C. we have, during the investigation, continually compared the E. M. F. of the two WESTON-standards used with that of a CLARK-cell, which was placed in the same thermostat as the WESTON-cell at 25° 0 C. We found:

Date	$\left(\frac{\text{E.M.F. CLARK}}{\text{E.M. WESTON}} \right)_{25^{\circ} 0}$
2 April 1924	1.3947
11 " "	1.3947
17 " "	1.3946
25 " "	1.3947
5 May "	1.3947
17 " "	1.3948
26 " "	1.3947
2 June "	1.3948
13 " "	1.3948

20. In connexion with the investigations of ERNST COHEN and A. L. TH. MOESVELD on the intermediate heats of solution of m. dinitro benzene in ethylacetate¹⁾, we have first of all tried to represent the relation between the intermediate heats of solution (Q_i) and the mean concentration (c) of the solutions of ammonium nitrate by means of a quadratic equation of the form:

$$Q_i = A + B(1 - c)^2,$$

in which A and B are constants, c the concentration expressed in gm. of salt per 100 gm. of solution. As this equation did not hold²⁾ along the great concentration interval which was investigated by us, we have tried an equation of the third degree, computed by the method of least squares.

For modification IV it runs:

$$(Q_i)_{IV} = -5910 + 83.057 c - 0.73476 c^2 + 0.00224 c^3 \quad . \quad (1),$$

for modification III:

$$(Q_i)_{III} = -5510 + 83.057 c_1 - 0.73476 c_1^2 + 0.00224 c_1^3 \quad . \quad (2)$$

¹⁾ Verslag etc. **26**, 1553 (1918); Zeitschr. f. physik. Chemie **93**, 385 (1919).

²⁾ According to this equation:

$$(Q_i)_{IV} = -5822 + 73.697 c - 0.4649 c^2$$

and

$$(Q_i)_{III} = -5422 + 73.697 c_1 - 0.4649 c_1^2,$$

we find divergencies between the values calculated and found as high as 0.8 per cent.

From column 11 and 12 in Table II we see, that these equations satisfy the determinations.

21. If in (2) we put $c_1 = c$, we find for the heat of transition III \rightarrow IV (at 32° 3 C.):

$$W = (Q_i)_{III} - (Q_i)_{IV} = -5510 + 5910 = \\ = +400 \frac{\text{gm. calories}}{\text{mol.}} = +4.99 \frac{\text{gm. calories}}{\text{gm.}}$$

BRIDGMAN (see par. 1) has found for it

$$4.66 \frac{\text{gm. calories}}{\text{gm.}}$$

As was to be expected according to our determinations of $(v_{III} - v_{IV})$ at 32° 3 we find in the caloric figure of BRIDGMAN an error of about the same magnitude as in his figure of volume measurements. It may be expected, and this was already pointed out in par. 1, that his determination of $\frac{dT}{dp}$ does not contain grave errors, because, owing to his method of procedure, the presence of metastable mixtures does not play a role here.

22. Finally a single remark on other investigations concerning the transformation of ammonium nitrate III into IV.

To an accidental compensation of errors must, in view of the facts mentioned above (par. 2 and 3), be attributed that BELLATI and ROMANESE have found a value for the heat of transition which is in close agreement to our figure. In this connexion we call attention to what ERNST COHEN and A. L. TH. MOESVELD¹⁾ communicated about the experimental testing of BRAUN's Law, in which investigation they proved that SILL's²⁾ results agreed with the equation mentioned to within a few per cents, notwithstanding the errors up to 20 per cent in his measurements.

We need not discuss here the determinations of TAMMANN³⁾ and of LUSSANA⁴⁾ as they lie outside the scope of these investigations, especially because an accurate measurement of pressure is wanting.

SUMMARY.

Intermediate heats of solution of the modifications III and IV of ammonium nitrate were determined at the transition temperature (32° 3 C.)

¹⁾ Verslag etc. **26**, 1241 (1918); Zeitschr. f. physik. Chemie **93**, 385 (1919).

²⁾ Journ. Americ. Chem. Soc. **38**, 2632 (1916).

³⁾ Kristallisieren und Schmelzen, Leipzig 1903, Page 299 et seq.

⁴⁾ Il Nuovo Cimento (4) **1**, 97 (1895).

by an adiabatic, electrical method. From these the heat of transition of this transformation was calculated. It was shown to be $4.99 \frac{\text{gm. cal.}}{\text{gm.}}$. As was to be expected, the value which BRIDGMAN found for it $\left(4.66 \frac{\text{gm. cal.}}{\text{gm.}}\right)$ is erroneous, as is also his determination of the change of volume at the transition temperature, in consequence of the fact that he experimented with a metastable mixture instead of with the pure modifications.

We beg to express our best thanks to Mr. H. G. S. SNIJDER, chem. docts., who very kindly assisted us in carrying out the caloric measurements.

Utrecht, June 1924.

VAN 'T HOFF-Laboratory.

Mathematics. — "*The Rank-Numbers of an Involution of Rays in Space*". By Dr. G. SCHAAKE. (Communicated by Prof. HENDRIK DE VRIES).

(Communicated at the meeting of October 25, 1924).

§ 1. Whereas involutions of groups of points in the plane or in space have already been investigated extensively a long time ago, the involutions of rays in space have only recently drawn the attention, except the involutions of the pairs of associated directrices of a linear complex and of the pairs of mutual polar lines of a quadratic surface. Prof. JAN DE VRIES has indicated and investigated several of those involutions ¹⁾ and Prof. C. H. VAN OS has treated another ²⁾.

For the investigation of the involutions of points in the plane the introduction of the rank-number, which indicates how many pairs of points of such an involution lie on an arbitrary straight line, has been of great importance. If r is this rank-number for an involution of pairs of points of the order n in the plane, this involution has a curve of double points of the order $n-2r$. BERTINI ³⁾, MARTINETTI ⁴⁾ and BERZOLARI ⁵⁾ have determined the involutions of pairs of points for which the value of this rank-number is from one to five. The involutions of point-triples for which this rank-number is one, have been investigated by Prof. JAN DE VRIES ⁶⁾.

In the following paper we shall show that to an involution of rays of which two conjugated lines cross each other as a rule, there belong *three* rank-numbers. After that we shall derive some relations existing between these rank-numbers, the order of the complex of the double-lines and a few other numbers that are characteristic for the involution of rays. Finally we shall try to find which are the involutions of rays of which the rank-numbers have the values zero or one.

§ 2. If we represent the rays of space in the well-known way on the points of a four-dimensional quadratic variety V_4 , lying in a five-

¹⁾ These Proceedings, 22, 478, 482, 634; 23, 462, 466; 22, 493; Revista Matematica, t. 5, p. 65. A paper of the author in these Proceedings, 22, 488 deals also with the involution of rays of the first mentioned paper.

²⁾ These Proceedings 22, 580.

³⁾ Lomb. Ist. Rend., ser. 2, vol. 16, p. 89 and 190.

⁴⁾ Annali di Mat., ser. 2, t. 12, p. 73, and ser. 2, t. 13, p. 53.

⁵⁾ Annali di Mat., ser. 2, t. 16, p. 191.

⁶⁾ These Proceedings, 16, 974 and 21, 291. Involutions of groups of n points have been treated by FERRETTI. Rend. di Palermo, t. 17, p. 311.

if m represents the number of straight lines which are conjugated in I to a given straight line.

The second rank-number r_2 of I indicates how many pairs of I lie in a given linear complex C and belong to one scroll together with two straight lines c_1 and c_2 given in C . If for c_1 and c_2 we choose two lines of C cutting each other, a scroll containing c_1 and c_2 always degenerates into the plane pencil (c_1, c_2) and another plane pencil w of which the vertex lies in the plane through c_1 and c_2 and the plane passes through the point of intersection of c_1 and c_2 . If (c_1, c_2) is chosen arbitrarily, there are apparently no two straight lines conjugated to each other in I which lie in this plane pencil. If n is the order of the scroll corresponding in I to an arbitrary plane pencil, C has n straight lines in common with the scroll conjugated to (c_1, c_2) . If further ε is the order of the complex of the double lines of I , the number $n - \varepsilon$ of the pairs of I belonging to C of which one straight line lies in (c_1, c_2) , is included in r_2 . We exclude the ε double rays of I in (c_1, c_2) through which the scroll corresponding to (c_1, c_2) also passes, because as a rule the carriers of the coincidences of the point-involution on V_4 representing I , which correspond to these double rays, do not lie in the R_4 which is conjugated to C .

r_2 includes also the number of the pairs of rays of I in plane pencils of C the vertices of which lie in the plane through c_1 and c_2 and the planes of which pass through the point of intersection of these lines. The plane pencils of C satisfying these two conditions, form a pencil of ∞^2 individuals for which the three characteristic numbers are equal to one. For the characteristic numbers of such a system indicate the order of the surface of the vertices, the class of the envelope of the planes, and the number of plane pencils of which the planes pass through a given point and the vertices lie in a given plane. This system has $\alpha + \beta + \gamma$ individuals in common with the system (α, β, γ) of ∞^3 plane pencils containing two intersecting rays of I).

Hence:

$$r_2 = n - \varepsilon + \alpha + \beta + \gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The order of the complex of the double lines of an involution of rays is accordingly equal to:

$$n - r_2 + \alpha + \beta + \gamma.$$

Finally we treat in the same way the third rank-number r_3 of I , which indicates how many pairs of rays of I belong to a given linear congruence. We choose a bilinear congruence of which the directrices cut each other and which is accordingly formed by all the straight lines passing through the point of intersection P of the directrices or lying in the plane π of these lines. We see in this case that r_3 contains in the

¹⁾ See e.g. my thesis for the doctorate: "Afbeeldingen van figuren op de punten eener lineaire ruimte", Groningen 1922, p. 112.

first place the numbers α and β of the pairs of I of which the lines resp. cut each other in P or lie in π . If p is the number of pairs of I of which one line passes through a given point and the other lies in given plane, r_3 contains also the number $p-\varepsilon$ of the pairs of I of which one line passes through P and the other lies in π . We exclude the ε coincidences of I in the plane pencil (P, π) , because the carriers of the corresponding coincidences of the point-involution conjugated to I on V_4 , do not lie in the R_3 which contains the image of the chosen degenerate bilinear congruence.

We find accordingly for r_3 :

$$r_3 = p - \varepsilon + \alpha + \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

so that the order of the complex of the double lines of I may also be written :

$$p - r_3 + \alpha + \beta.$$

By the aid of (1) and (2) we arrive at the following relation, which exists between m , n , p and the rank-numbers of I :

$$r_1 - r_2 + r_3 = m - n + p \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We find a formula containing another number remarkable for I , if we consider the correspondence which arises if we conjugate to each line l of a given plane pencil the line l'' of the same pencil which cuts a line l' associated to l in I . In this correspondence apparently m lines l' are associated to a line l . As besides ε generatrices the scroll corresponding to the plane pencil has a curve of the order $n-\varepsilon$ in common with the plane of the pencil, a ray l'' cuts $n-\varepsilon$ more lines l' and consequently $n-\varepsilon$ lines l correspond to a line l'' . Accordingly the correspondence (l, l') has $m + n - \varepsilon$ coincidences. These lie in the first place in the ε double rays of I in the given plane pencil and further in the σ generatrices each of which is cut by a line conjugated to it in I . The number of the latter generatrices may therefore be found by the formula:

$$\sigma = m + n - 2\varepsilon^1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The number σ may also be considered as the order of the complex of the lines that are cut by the lines which are conjugated to them in I .

We remark that the formulas derived in this §, cannot be applied to an involution of rays which only consists of pairs of intersecting lines. For if in this case we conjugate to each other the point of intersection and the plane of any such a pair of lines, we do not get a null-system but a system of ∞^4 plane-elements and accordingly α, β and γ as well as σ become infinite.

The considerations of § 1—§ 3 hold also good if instead of an involution we take an involutorial correspondence in the rays of space.

§ 4. From formula (1) of § 3 there follows that the first rank-number of an involution of rays I in which to each straight line of space one or more straight lines are conjugated, is at least one. This appears also

¹⁾ Cf. SCHUBERT, Kalkül der Abzählenden Geometrie, p. 58, (5).

in the following way. If the first rank-number is zero, no straight line of the system S_4 of ∞^4 rays corresponding to I passes through an arbitrary point of R_5 . The lines of S_4 form a variety which does not coincide with V_4 , because in that case all the pairs of I would consist of lines which cut each other. Accordingly straight lines of S_4 pass only through those points of V_4 which lie on the intersection of V_4 and the variety containing S_4 , so that no line in I corresponds to a line of which the image point does not lie on this intersection.

We shall therefore investigate only those involutions of rays for which the first rank-number is one:

If the three rank-numbers are one, zero and zero, the carriers of the pairs of the point-involution on V_4 corresponding to I , form a system S_4 (1,0,0) of ∞^4 straight lines of R_5 . According to § 2 an arbitrary R_3 does not contain any line of S_4 , nor does an arbitrary R_4 contain a straight line which cuts another straight line chosen arbitrarily in this R_4 . Hence an arbitrary R_4 does not contain any line of S_4 . For the lines of S_4 in an R_4 would form one or more surfaces each containing ∞^2 straight lines, as a line of R_4 which cuts one line of S_4 in this space, must cut an infinite number of them, for which reason an R_4 cannot contain a system of ∞^1 or a finite number of lines of S_4 . This surface would necessarily be formed by one or more planes, which is in contradiction to the fact that an R_3 , through which we can always pass an R_4 , does not contain any line of S_4 .

The four-dimensional spaces R_4^1 which contain straight lines of S_4 , form therefore a special system. One of the individuals of this system passes through a given R_3 . For the rays of S_4 cutting an R_3 , form a variety which has in common with an R_4 through this R_3 the latter space, because, as appears from the first characteristic number of S_4 , through any point of the R_3 there passes a line of S_4 . As the R_4 mentioned does not generally contain any line of S_4 , the lines of S_4 which cut an R_3 , lie in a linear four-dimensional space, the only R_4 which may be passed through this R_3 in which there lie lines of S_4 . The spaces R_4 are accordingly the four-dimensional spaces that pass through a point S of R_5 and the rays of S_4 are the lines through S . Two points of V_4 on the same line of S_4 represent two rays which are conjugated to each other relative to the linear complex corresponding to the intersection of V_4 with the polar space of S relative to V_4 .

Consequently an involution of rays with rank-numbers one, zero and zero is always formed by the pairs of lines which are conjugated to each other relative to a linear complex.

For such an involution $m = n = p = \varepsilon = 1$ and $\alpha = \beta = \gamma = \sigma = 0$. These numbers satisfy the five formulas found in § 3.

To an involution of rays for which the rank-numbers are one, zero and one, there corresponds on V_4 an involution of pairs of points the carriers of which form a system S_4 (1, 0, 1) of ∞^4 rays of R_5 . In this

case the lines of S_4 which belong to an R_4 , are the rays of a congruence $(0,1)$, that is a system of ∞^2 straight lines none of which cuts a line given in this R_4 and one of which belongs to an R_3 given in the R_4 . Accordingly the lines of this congruence form a surface which must be one plane V , because any line of the R_4 passing through a point of this surface, must cut an infinite number of lines of the congruence and an R_3 of the R_4 contains one line of the congruence. As through a plane there pass ∞^2 four-dimensional spaces, there are ∞^3 planes V .

Now through an arbitrary point of R_5 there passes only one line of S_4 . Hence all the straight lines of S_4 cannot lie in planes V .

We consider two planes V , which we call V_1 and V_2 . They have one line in common, viz. the line of S_4 lying in the R_3 which belongs to an R_4 through V_1 and an R_4 through V_2 . In the same way the plane V of an arbitrary R_4 has one line in common with each of the planes V_1 and V_2 and it belongs therefore to the R_3 which is defined by V_1 and V_2 . Consequently the system S_4 degenerates into that of the lines of the latter three-dimensional space R_3^1 and a system with rank-numbers one, zero and zero which is formed by the lines of R_5 passing through a definite point. Accordingly the involution of rays consists of the ∞^4 pairs of straight lines that are conjugated to each other relative to a linear complex, and of the ∞^4 pairs of rays of the bilinear congruence that are represented on the intersection of R_3^1 and V_4 .

There is no non-degenerate involution of rays of which the rank-numbers are one, zero and one.

§ 5. Now we shall investigate the involution of rays for which the three rank-numbers are one, one and zero, which is represented on a point-involution of V_4 of which the system of carriers is a system S_4 $(1,1,0)$ of ∞^4 lines in R_5 . The locus of the ∞^2 rays of S_4 lying in a four-dimensional space R_4 of R_5 , one of which cuts a line given in R_4 as appears from the second rank-number of S_4 , is a linear three-dimensional space R_3^1 . In an arbitrary R_3 of R_4 there lies no line of S_4 , hence neither in the plane of intersection of this R_3 and the space R_3^1 that corresponds to R_4 . The lines of R_3^1 which belong to an R_4 , form therefore a congruence $(1,0)$, that is a sheaf of rays, of a three-dimensional space R_3^1 of this R_4 .

If an R_4 describes a pencil of spaces, the locus of the vertices of the sheaves of rays of S_4 which lie in the spaces of this pencil, has no point in common with the three-dimensional space of the axes, because the sheaf of rays of S_4 which has such a point as vertex, would have a straight line [in the space of the axes and this space generally does not contain any line of S_4 . The locus of the vertices of the sheaves of S_4 in the spaces of the pencil under consideration has outside the space of the axes one point in common with each of its individuals and it is therefore a line a .

An arbitrary R_4 cuts a in a point A and has in common with the R_4 that belongs to the above mentioned pencil and passes through A , a plane pencil that has A as vertex and the plane of which is the intersection of the former R_4 and the R_3^1 of the latter R_4 . Hence the vertex of the sheaf of rays of S_4 that lies in an arbitrary R_4 , always belongs to a and all the lines of S_4 cut the line a . As an R_4 through a point A of a has a sheaf of rays in a three-dimensional space in common with the locus of the lines of S_4 through A , this locus is a linear four-dimensional space through A .

Accordingly to each point A of a there corresponds a four-dimensional space R_4^1 so that the lines of this space through A are the rays of S_4 through A . There are ∞^1 spaces R_4^1 , one of which passes through an arbitrary point P of R_5 , namely the R_4^1 that is conjugated to the point of a where the line of S_4 through P cuts a . The spaces R_4^1 form therefore a pencil that has a three-dimensional space Γ as space of the axes; hence all the lines of S_4 cut also Γ .

Now the lines of intersection of a and Γ form just such a system $S_4(1, 1, 0)$. Any line cutting a and Γ must also belong to the chosen system S_4 because it is the only line cutting a and Γ which passes through one of its points outside a and Γ .

A system $S_4(1, 1, 0)$ of ∞^4 rays of R_5 consists therefore of the lines which cut a given line a and a likewise given linear three-dimensional space Γ^1).

The point L' corresponding to a given point L of V_4 in the point-involution defined on this variety by S_4 , is the second point of intersection of V_4 with the line of intersection through L of the plane (L, a) and the four-dimensional space (L, Γ) . To the planes through a there correspond in the rays of space the scrolls containing the two lines m and n the images of which are the points of intersection of a and V_4 and to the four-dimensional spaces through Γ there correspond the linear complexes containing the bilinear congruence which is represented on the intersection of Γ with V_4 .

An involution of rays for which the three rank-numbers are one, one and zero, is accordingly defined by two given straight lines m and n and a given bilinear congruence K . The line l' conjugated to a line l , is the line different from l which has the scroll (l, m, n) in common with the linear complex containing l and K .

By the aid of a representation of the rays of space on a linear four-dimensional space of points, this involution has been treated in p. 84—92 of the thesis for the doctorate to which we referred in the foot-note to § 3.

To a plane pencil, a sheaf, and a field of lines l there are resp. conjugated a cubic scroll, a congruence $(1, 2)$ and a congruence $(2, 1)$ of lines

¹⁾ a can be infinitely near to Γ .

l' . Hence the numbers m , n and p of § 3 are resp. 1, 3 and 2 so that the relation (4) of § 3 is satisfied.

As this involution of rays has a quadratic complex of double lines, there results from the formulas (1), (2), (3) and (5) of § 3 that $\alpha = \beta = \gamma = \sigma = 0$. The ∞^3 pairs of rays (l, l') for which l and l' cut each other, are in fact the pairs of generatrices of the plane pencils that contain m or n and a line of K , and these pairs (l, l') define only ∞^1 plane-elements; the lines l that are cut by the conjugated line l' , form no complex but two special bilinear congruences.

§ 6. Finally we determine the involution of rays for which each of the three rank-numbers is one. Such an involution I is represented on a point-involution of V of which the carriers of the pairs of points form a system $S_4(1, 1, 1)$ of ∞^4 lines in R_5 . According to § 2 one line d of S_4 passes through a given point P of R_5 . The rays of S_4 which lie in an R_4 , are the lines of a system of ∞^2 lines one of which cuts a given straight line of this R_4 ; they form therefore a linear three-dimensional space R_3^1 . An arbitrary three-dimensional space R_3 of the chosen R_4 contains one line of S_4 , which must lie in the plane of intersection of R_3 and R_3^1 .

Hence we see that an R_4 contains a bilinear congruence of S_4 lying in a three-dimensional space R_3^1 that belongs to the R_4 ¹⁾.

As each R_4 contains one R_3^1 , but each R_3^1 is conjugated to the ∞^1 spaces R_4 passing through it, there are ∞^4 spaces R_3^1 each of which contains a bilinear congruence of S_4 . The three-dimensional space through two crossing lines d of S_4 contains more than one, hence an infinite number of lines of S_4 and must be conjugated as R_3^1 to each four-dimensional space passing through it. If two spaces R_3^1 have only one line in common, this is a generatrix of S_4 . For if we pass an R_3 through each of these spaces, they have an R_3 in common and the line of S_4 belonging to this R_3 , must necessarily be the line of intersection mentioned.

We choose a space R_3^1 , which we call Δ , and a straight line d_1 of S_4 which does not cut R_3^1 . In order to do this we have only to start from an arbitrary R_4 and to choose the R_3^1 belonging to it with the line d_1 of S_4 through an arbitrary point of the R_4 outside this R_3^1 . A space R_3^1 through d_1 cuts Δ along a line of the bilinear congruence C_1 of S_4 lying in Δ . Inversely an R_3 through d_1 and a straight line of C_1 is a space R_3^1 . Hence the projection of S_4 out of d_1 on Δ is C_1 , the same as that of a bilinear congruence of C of S_4 which lies in a space R_3^1 that does not pass through d_1 .

If δ_1 and δ_2 are the directrices of C , one of the directrices of each

¹⁾ If the lines of an R_4 always formed a sheaf of rays in an R_3 and a field, S_4 would degenerate into a system $(1,1,0)$ and a system $(0,0,1)$.

congruence C lies in the three-dimensional space (d_1, δ_1) , the other in (d_2, δ_2) . One directrix of C must apparently lie in both the spaces (d_1, δ_1) and (d_2, δ_2) , which, as they do not coincide and contain moreover both the line δ_1 , have a plane V_1 in common in which each congruence C has one of its directrices. In the same way it appears that the other directrix of each congruence C lies in a plane V_2 which the spaces (d_1, δ_2) and (d_2, δ_2) have in common.

Accordingly the directrices of S_4 must cut the two planes V_1 and V_2 . Now the straight lines which cut two planes V_1 and V_2 chosen at random in R_5 , form a system $S_4(1, 1, 1)$. Each line cutting V_1 and V_2 must also belong to the chosen system S_4 , because it is the only line cutting V_1 and V_2 which passes through one of its points outside V_1 and V_2 .

Consequently a system of ∞^4 rays in R_5 of which the three characteristic numbers are one, always consists of the lines which cut two given planes ¹⁾.

We find the point L^1 corresponding to a given point L of V_4 in the point-involution on this variety where the lines of S_4 are the carriers of the pairs of points, if we form the three-dimensional spaces (L, V_1) and (L, V_2) and determine the second point of intersection with V_4 of the line of intersection of these spaces. To V_1 and V_2 there correspond in the rays of space resp. the scrolls φ_1 and φ_2 . In order to find the ray l' which corresponds in the involution in question to a given line l , we must therefore form the two bilinear congruences (l, φ_1) and (l, φ_2) . l' is the line which these congruences have in common besides l . The pairs of directrices of (l, φ_1) and (l, φ_2) are resp. the pairs of generatrices of the scrolls ϱ_1 and ϱ_2 conjugated to φ_1 and φ_2 which cut l .

Accordingly an involution of rays of which the three rank-numbers are one, always consists of the pairs of rays of which the lines l and l' cut the same generatrices of two given scrolls ϱ_1 and ϱ_2 .

This involution has already been investigated by Prof. JAN DE VRIES ²⁾.

To a plane pencil, a sheaf and a field of lines l , there correspond resp. a cubic scroll, a congruence $(2, 3)$ and a congruence $(3, 2)$ of lines l' . Hence the numbers m, n and p of § 3 are resp. 1, 3 and 3. The relation (4) of § 3 is accordingly satisfied.

The formula (1) of § 3 gives that $\gamma = 0$. If we have two conjugated lines l and l' of the involution in question which cut each other, through the point of intersection there must pass a generatrix of ϱ_1 and a generatrix of ϱ_2 and the plane through l and l' must contain a line of each of these two scrolls. Hence the point of intersection of two such lines l and l' always lies on the curve of intersection k^4 of the two quadratic surfaces formed by ϱ_1 and ϱ_2 , and the plane (l, l') is a common tangent

¹⁾ V_1 and V_2 can be infinitely near to each other.

²⁾ These Proceedings 22, 634. By the aid of the representation of the rays of space on V_4 , this involution has been treated by Mr. H. B. BONE, Wiskundige Opgaven, part 14, p. 150.

plane of them. Any point of k^4 is the vertex of one plane pencil of which two arbitrary generatrices form a pair of the involution in question. Hence $\alpha = \beta = \gamma = 0$. As the lines l that are cut by the conjugated lines l' , form a congruence, of which an arbitrary plane pencil does not contain any line, also $\sigma = 0$. Now the formulas (2), (3) and (5) of § 3 give, that there exists a quadratic complex of double rays, which has indeed been found.

If V_1 and V_2 have a point in common, the same holds good for the planes W_1 and W_2 conjugated to them relative to V_4 , and φ_1 and φ_2 lie in the linear complex K_φ that is conjugated to the space (V_1, V_2) , and ϱ_1 and ϱ_2 in the linear complex K_φ that corresponds to the space (W_1, W_2) . In this case the system S_4 of the lines cutting V_1 and V_2 degenerates into the system of lines through the point of intersection of V_1 and V_2 and the system of lines of the four-dimensional space (V_1, V_2) which cuts the planes V_1 and V_2 . The rays of the latter system are the lines of the ∞^2 planes which have a line in common with each of the planes V_1 and V_2 and they are accordingly the carriers of the pairs of points of the ∞^2 conics of V_4 that have two points in common with each of the conics along which V_4 is intersected by V_1 and V_2 . The involution of rays corresponding to S_4 , degenerates in this case into that of the pairs of rays which are conjugated to each other relative to K_φ and the system of the ∞^4 pairs of rays of the ∞^2 scrolls which contain two lines of each of the scrolls φ_1 and φ_2 . The rank-numbers of the latter system are zero, one and one.

If φ_1 and φ_2 are the scrolls of the same quadratic surface Φ and accordingly ϱ_1 and ϱ_2 coincide resp. with φ_2 and φ_1 , the planes V_1 and V_2 are conjugated to each other relative to the variety V_4 . I splits off from the system of the ∞^4 pairs of lines which touch Φ at the same point.

The remaining involution, of which still all the rank-numbers are one, is the involution of the pairs of mutual polar lines of the quadratic surface Φ .

For this involution of rays $m=n=p=1$. Further $\alpha=\beta=\gamma=0$, because two intersecting straight lines are conjugated to each other if they belong to the same plane pencil of tangents to Φ and if they separate harmonically the two lines of Φ lying in this plane pencil. The formulas (1) and (4) of § 3 are therefore satisfied. There follows from (2) and (3) that $\varepsilon=0$. There are in fact in this case only two scrolls of double lines. Further (5) gives that $\sigma=2$. The lines in a plane pencil that are cut by the conjugated lines, are accordingly the tangents to Φ lying in the plane pencil.

Physics. — “*Determinations of the Terms in the Lanthanum Spectrum*” ¹⁾.

By S. GOUDSMIT. (Communicated by Prof. P. EHRENFEST).

(Communicated at the meeting of October 25, 1924).

Most earlier measurements of the ZEEMAN effect in not classified spectra are insufficient to successfully apply LANDÉ's rules ²⁾ to them with a view to arrange the lines. An exception to this are the accurate measurements by S. RYBAR ³⁾ in the spectrum of Lanthanum.

In this same spectrum groups of lines with equal frequency differences have been found by E. PAULSON ⁴⁾.

Starting from these investigations a number of lines could be arranged in a term system, the relative term values could be determined, and the corresponding “innere Quantenzahl” J and factor of resolution g could be found for every term. Of the groups of lines found by PAULSON only those have been taken which were sufficiently certain by the observed ZEEMAN effects. Also several other lines have been classified, which did not occur in PAULSON's groups of lines.

From the ZEEMAN resolutions it appeared first of all that all classified lines belong to odd term-systems, hence that according to KOSSEL and SOMMERFELD's law of displacement they all are due to the *ionized* Lanthanum atom.

When the “innere Quantenzahl” J and the resolution factor g of a term are known from the ZEEMAN effects, it is generally possible to determine also the quanta values K and R by the aid of LANDÉ's formulae or table, i.e. the term symbol s, p, d etc. and also the term system, doublet, triplet, etc. can be found.

For some terms found in the Lanthanum spectrum this does not apply, however. Evidently these are the terms “höherer Stufe” examined of late by LANDÉ and HEISENBERG ⁵⁾. Of these terms the relation between g and the quanta values is not yet known, hence their term symbols could not be determined. They are indicated by Roman numerals in the tables.

Nor is it excluded that some of the ordinary terms of the first rank found are in reality terms of higher rank, which happen to have

¹⁾ Preliminary communication in *Die Naturwissenschaften*. 12, 851, 1924.

²⁾ A. LANDÉ, *Zeitschr. f. Phys.* 15, 189, 1923.

³⁾ S. RYBAR, *Diss. Budapest and Phys. Zeit.* 12, 889, 1911.

⁴⁾ E. PAULSON, *Ann. der Phys.* 45, 1203, 1914.

⁵⁾ W. HEISENBERG, *Zeitschr. f. Phys.* 26, 291, 1924. A. LANDÉ u. W. HEISENBERG, *Zeitschr. f. Phys.* 25, 279, 1924.

corresponding J and g values. This might e.g. be the case with the triplet terms \overline{d}_1 and \overline{d}_2 , as the corresponding term \overline{d}_3 could not be found.

The occurrence of such terms of higher rank means, according to LANDÉ, that in the state in question the atom rest — i.e. the atom without the emitting electron — contains electron groups with azimuthal quanta values greater than 1, which do not form a closed configuration without moment of momentum. In the case of ionized Lanthanum this means very probably that the non-emitting electron of the two outer ones can sometimes describe a 6_1 - and sometimes a 5_3 -path. In the former case the ordinary terms are obtained, in the latter case those of higher rank.

TERM TABLE.

Relative value of the term	Term symbol	J	g
0	p_1	$2\frac{1}{2}$	$\frac{3}{2}$
1043.4	p_2	$1\frac{1}{2}$	$\frac{3}{2}$
1418.8	p_3	$\frac{1}{2}$	$0/0$
3705.8	I	$2\frac{1}{2}$	1.50 ± 0.01 (tripl. p_1 of quint. d_3 ?)
4888.7	\overline{d}_1	$3\frac{1}{2}$	$\frac{4}{3}$
5049.5	II	$1\frac{1}{2}$	1.28 ± 0.03
5780.3	III	$1\frac{1}{2}$	0.87 ± 0.01
5815.8	\overline{d}_2	$2\frac{1}{2}$	$\frac{7}{6}$
6790.1	IV	$2\frac{1}{2}$	0.84 ± 0.04
7231.0	V	$1\frac{1}{2}$	0.78 ± 0.02
8681.5	VI	$3\frac{1}{2}$	1.05 ± 0.03 (tripl. t_2 ?)
8741.4	VII	$2\frac{1}{2}$	0.88 ± 0.04
26975.5	\overline{p}_1	$2\frac{1}{2}$	$\frac{3}{2}$
27484.7	\overline{p}_2	$1\frac{1}{2}$	$\frac{3}{2}$
27953.1	\overline{p}_3	$\frac{1}{2}$	$0/0$
29952.4	d_1	$3\frac{1}{2}$	$\frac{4}{3}$
30611.6	d_2	$2\frac{1}{2}$	$\frac{7}{6}$
31308.6	d_3	$1\frac{1}{2}$	$\frac{1}{2}$
31808.4	D	$2\frac{1}{2}$	1
33202.8	VIII	$2\frac{1}{2}$	0.73 ± 0.02

THE COMBINATION $p\bar{p}$

λ	I	$\nu_{vac.}$	ZEEMAN effect	
			observed	calculated
3855.10	3	25932.3	(0) 1.49	(0) 1.50
3835.29	5	26066.3	(0) 1.51	(0) 1.50
3780.85	3 _r	26441.6	(0) 1.40	(0) 1.50
3715.03	3	26910.0	(0) 1.53	(0) 1.50
3706.02	5	26975.5	(0) 1.50	(0) 1.50
3637.35	3	27484.7	(0) 1.50	(0) 1.50

	$p_1 \ 2\frac{1}{2}$		$p_2 \ 1\frac{1}{2}$		$p_3 \ 1\frac{1}{2}$
$\bar{p}_1 \ 2\frac{1}{2}$	26975.5	1043.2	25932.3		
	509.2		509.3		
$\bar{p}_2 \ 1\frac{1}{2}$	27484.7	1043.1	26441.6	375.3	26066.3
			468.6		
$\bar{p}_3 \ 1\frac{1}{2}$			26910.0		

THE COMBINATION $p d$

λ	I	$\nu_{vac.}$	ZEEMAN effect	
			observed	calculated
3381.10	10	29567.7	(0) (0.40) 0.82 1.17 1.59	(0) (0.33) 0.83 1.17 1.50
3344.71	7	29889.4	(0) 0.52	(0) 0.50
3337.67	15	29952.4	(0) 1.13	(0) 1.00 1.17 1.33.....
3303.26	5	30264.4	0.52 (1.00) 1.53	(0.50) (1.00) 1.53
3265.79	4	30611.6	(0.67) 0.76 1.15 1.59 1.97	(0.67) 0.83 1.17 1.50 1.83
3193.09	1	31308.6	(0) (1.04) 1.42? 2.50	(0) (1.00) 1.50 2.50

	$p_1 \ 2\frac{1}{2}$		$p_2 \ 1\frac{1}{2}$		$p_3 \ 1\frac{1}{2}$
$d_1 \ 3\frac{1}{2}$	29952.4				
	659.2				
$d_2 \ 2\frac{1}{2}$	30611.6	1043.9	29567.7		
	697.0		696.7		
$d_3 \ 1\frac{1}{2}$	31308.6	1044.2	30264.4	375.0	29889.4

It is seen in these groups that the ratios of the term differences depart greatly from the interval rule of LANDÉ. They also give a good idea of the agreement between observed and calculated ZEEMAN effects.

It is remarkable that in his interesting paper on triplet combinations S. POPOW¹⁾ already suspected that the last group was a triplet pd combination.

In the calculation of the relative term-values the term p_1 was put equal to 0; it is, however, uncertain whether this term represents the highest or the lowest of the levels of energy.

TABLE A. OTHER COMBINATIONS.

Combination	λ	I	$\nu_{vac.}$	ZEEMAN effect	
				observed	calculated
$\overline{p}_1 \ 2^{1/2}$ with					
I $2^{1/2}$	4296.21	8r	23269.8 (0)	1.49	(0) 1.50
$\overline{d}_1 \ 3^{1/2}$	4526.30	5	22086.9 (0)	1.11	(0) 1.00 1.17 1.33...
II $1^{1/2}$	4559.51	2	21926.0 (0)	1.67	(0) 1.72
III $1^{1/2}$	4716.59	(8)	21195.8 (0)	(0.65) 0.86 1.45 2.13	(0) (0.63) 0.85 1.50 2.13
$\overline{d}_2 \ 2^{1/2}$	4724.57	(5)	21160.0	—	—
IV $2^{1/2} ?$	4952.21	(4)	20187.4 (0)	1.21	0.84 (1.32) 1.50
V $1^{1/2}$	5063.07	(2)	19745.4	—	—
VI $3^{1/2}$	5464.57	(5)	18294.6	—	—
VII $2^{1/2}$	5482.47	(6)	18255.0	—	—
$\overline{p}_2 \ 1^{1/2}$ with					
I $2^{1/2}$	4204.23	4r	23778.9 (0)	1.48	(0) 1.50
II $1^{1/2}$	4455.99	2	22435.4 (0.20)	1.40	(0.22) 1.28 1.50
III $1^{1/2}$	4605.99	2	21704.8 (0.62)	0.85 1.51	(0.63) 0.87 1.50
$\overline{d}_3 \ 2^{1/2}$	4613.57	4	21669.1	—	—
IV $2^{1/2} ?$	4830.68	(2)	20695.2 (0)	0.76	(0) 0.18 (0.66) 0.84 1.50
V $1^{1/2}$	4935.77	(2)	20254.6	—	—
VII $2^{1/2}$?				
$\overline{p}_3 \ 1^{1/2}$ with					
II $1^{1/2}$	4364.84	3	22903.9 (0)	1.27	(0) 1.28
III $1^{1/2}$?				
V $1^{1/2}$	4824.22	(8)	20723.0 (0)	0.78	(0) 0.78

¹⁾ S. POPOW, Ann. der Phys. 45, 147, 1914.

The calculated resolutions of the terms treated above were the starting-point for the calculation of the other term resolutions of the combinations following. As these resolutions could be calculated from different combi-

TABLE A (continued).

Combination	λ	I	$\nu_{\text{vac.}}$	ZEEMAN effect	
				observed	calculated
d₁ 3¹/₂ with					
I 2 ¹ / ₂	3808.89	2	26246.9	(0) 1.20	(0) 1.00 1.17 1.33...
$\bar{\text{d}}_1$ 3 ¹ / ₂	3988.66	30	25064.0	(0) 1.32	(0) 1.33
$\bar{\text{d}}_2$ 2 ¹ / ₂	4141.90	10	24142.6	(0) 1.49	(0) 1.67 1.40
IV 2 ¹ / ₂	4316.07	1	23162.7	(0) (0.55) (1.10) 1.98 2.35	(0) (0.50) (1.00) 1.55 2.35
VI 3 ¹ / ₂	4699.80	(2)	21271.6	—	—
VII 2 ¹ / ₂	4713.08	(8)	21211.6	(0) (0.47) (0.92) 1.83 2.26	(0) (0.45) (0.91) 1.79 2.24
d₂ 2¹/₂ with					
I 2 ¹ / ₂	3715.67	4	26905.3	(0.64) 0.81 1.15 1.51 1.78?	(0.67) 0.83 1.16 1.50 1.84
$\bar{\text{d}}_1$ 3 ¹ / ₂	3886.51	15	25722.7	(0) 1.53	(0) 1.67 1.40...
II 1 ¹ / ₂	3910.95	(2)	25562.0	(0) 1.05	(0) 1.05
III 1 ¹ / ₂	4026.03	4	24831.4	(0) 1.33	(0) 1.46 1.16...
$\bar{\text{d}}_2$ 2 ¹ / ₂	4031.86	7	24795.5	(0) 1.15	(0) 1.16
IV 2 ¹ / ₂	4196.74	10	23821.3	(0.39) 0.52 (0.60) 0.81 1.14 [1.41]	(0.32) 0.52 (0.64) 0.84 1.16 [1.49]
V 1 ¹ / ₂	4275.80	4	23380.9	(0) (0.36) 1.13 1.51	(0) (0.38) 1.16 1.55
VI 3 ¹ / ₂	4558.66	4	21930.2	(0) 0.93	(0) 0.82 0.94 1.05...
VII 2 ¹ / ₂	4571.14	(2u)	21870.4	(0.43) 1.00	(0.57) 0.88 1.16
d₃ 1¹/₂ with					
I 2 ¹ / ₂	3621.96	1	27601.4	(0) (1.06) 2.52	(0) (1.00) 2.50
II 1 ¹ / ₂	?				
III 1 ¹ / ₂	3916.21	10	25527.7	(0.38) 0.47 0.89	(0.37) 0.50 0.87
$\bar{\text{d}}_2$ 2 ¹ / ₂	3921.71	10	25491.9	(0) 0.57 (0.64) 1.19 1.82	(0) 0.50 (0.66) 1.16 1.87
IV 2 ¹ / ₂	4077.51	15	24517.9	(0) (0.37) 0.50 0.84 1.18	(0) (0.34) 0.50 0.84 1.18
V 1 ¹ / ₂	4152.17	10	24077.0	(0.29) 0.48 0.78	(0.28) 0.50 0.78
VII 2 ¹ / ₂	4430.11	8	22566.5	(0) (0.43)... 0 90? 1.34	(0) (0.38) 0.50 0.90 1.26

nations, the means of the g values found were given in the term table with the possible deviations. This was, of course, not necessary for the ordinary terms of the first rank of which the exact values of g have been given by LANDÉ.

Table A gives the other combinations between the terms given above with the observed and the calculated ZEEMAN effects. The latter have been calculated with the mean g -values given in the term-table.

TABLE A (continued).

Combination	λ	I	$\nu_{vac.}$	ZEEMAN effect	
				observed	calculated
D $2^{1/2}$ with					
p ₁ $2^{1/2}$	3142.99	1	31807.6	—	—
p ₂ $1^{1/2}$	3249.33	3	30765.1	(0) 0.47 (0.49) 1.02	(0) 0.50 (0.50) 1.00
I $2^{1/2}$	3557.40	1	28102.4	—	—
\bar{d}_1 $3^{1/2}$	3713.71	6	26919.6	(0) (0.32) (0.64) 1.72 2.06	(0) (0.33) (0.67) 1.67 2.00
II $1^{1/2}$	3736.02	5	26758.9	—	—
III $1^{1/2}$	3840.92	5	26028.0	(0) 1.05	(0) 1.13 1.00
\bar{d}_2 $2^{1/2}$	3846.16	3	25992.6	(0.28) 1.12	(0.33) 1.00 1.16
IV $2^{1/2}$	3995.91	5	25018.6	(0.27) 0.90	(0.32) 0.84 1.00
V $1^{1/2}$	4067.52	8	24578.1	(0) 1.09	(0) 1.22 1.00
VI $3^{1/2}$	4322.69	5	23127.3	(0) 1.08	(0) 1.15 1.10 1.05
VII $2^{1/2}$	4333.97	15	23067.0	(0.19) 0.95	(0.24) 0.88 1.00
VIII $2^{1/2}$ with					
p ₁ $2^{1/2}$	3010.92	(2)	33202.8	—	—
p ₂ $1^{1/2}$	3108.57	(2)	32159.8	—	—
I $2^{1/2}$?				
\bar{d}_1 $3^{1/2}$	3530.80	1	28274.1	—	—
II $1^{1/2}$	3550.98	1	28153.2	—	—
III $1^{1/2}$	3645.57	8	27422.7	(0) 0.63	(0) 0.59
\bar{d}_2 $2^{1/2}$	3650.31	4	27394.6	0.29 0.73 (0.87) 1.17 1.60	0.30 0.73 (0.87) 1.16 1.60
IV $2^{1/2}$	3784.95	2	26413.0	(0.17) 0.79	(0.22) 0.73 0.84
V $1^{1/2}$	3849.20	10	25972.1	(0) 0.69	(0) 0.68
VI $3^{1/2}$	4076.85	2	24521.8	(0) (0.35) (0.61) 1.33 1.67	(0) (0.32) (0.64) 1.37 1.69
VII $2^{1/2}$	4086.90	20	24461.5	(0.27) 0.81	(0.30) 0.73 0.88

TABLE B. LINES WITH KNOWN ZEEMAN EFFECT, WHICH HAVE NO PLACE AS YET IN THE TERM SYSTEM.

λ	I	$\nu_{vac.}$	ZEEMAN effect observed	Combination $J_1 \quad g_1$ with $J_2 \quad g_2$	
3104.76	1	32199.3	(0) (0.50) ... 1.49	$2\frac{1}{2}$ 1.00	$1\frac{1}{2}$ 0.50
3513.06	1	28457.1	0.54 (0.57) 1.08	$1\frac{1}{2}$ 0.54	$1\frac{1}{2}$ 1.08
3725.24	3	26836.3	(0) (0.39) (0.74) 1.47 1.82	$2\frac{1}{2}$ 0.73	$3\frac{1}{2}$ 1.10
4152.97	5	24072.4	0.32 0.64 (0.76) 1.05 1.42	$2\frac{1}{2}$ 0.66	$2\frac{1}{2}$ 1.05
4238.57	10	23586.2	(0.64) 0.73 1.00 1.30 1.60	$2\frac{1}{2}$ 1.00	$2\frac{1}{2}$ 1.30
4250.17	6	23521.8	(0.65) 0.66 0.98 1.30 1.60	$2\frac{1}{2}$ 1.00	$2\frac{1}{2}$ 1.32
4300.59	3	23246.1	0 0.73 (0.73) 1.46 (1.50) 2.18	$2\frac{1}{2}$ 0.73	$2\frac{1}{2}$ 1.46
4383.62	5	22805.8	(0) (0.42) (0.87) 1.50 1.89	$2\frac{1}{2}$ 0.64	$3\frac{1}{2}$ 1.06
4436.02	1	22536.4	(0) (0.57) (1.19) 1.80 2.43	$2\frac{1}{2}$ 0.60	$3\frac{1}{2}$ 1.20
4474.72	(3)	22341.5	(0) (0.76) 1.11 1.84	$2\frac{1}{2}$ 1.10	$1\frac{1}{2}$ 0.36
4575.08	4	21851.4	(0.48) 0.51 0.95 (0.98) 1.46 2.07	$2\frac{1}{2}$ 0.98	$2\frac{1}{2}$ 1.48
4647.64	1	21510.3	(0) (0.44) (0.86) 1.74 2.15	$2\frac{1}{2}$ 0.88	$3\frac{1}{2}$ 1.30
4669.10	3	21411.4	(0) (0.40) (0.72) 1.51 1.86	$2\frac{1}{2}$ 0.75	$3\frac{1}{2}$ 1.12
4728.55	(10)	21142.2	(0) (0.39) (1.59) 1.89	$2\frac{1}{2}$ 1.11	$1\frac{1}{2}$ 0.72
4804.22	(7)	20809.2	0.55 (0.93) 1.43	$1\frac{1}{2}$ 0.55	$1\frac{1}{2}$ 1.43
4946.60	(5)	20210.3	(0) (0.62) 1.17 1.81	$2\frac{1}{2}$ 1.17	$1\frac{1}{2}$ 0.54
4986.99	(6)	20046.6	(0) (0.46) 0.95 1.40	$2\frac{1}{2}$ 0.95	$1\frac{1}{2}$ 0.50

TABLE C. CLASSIFIED LINES.

λ	I	$\nu_{vac.}$	Combination	λ	I	$\nu_{vac.}$	Combination
3010.92	(2)	33202.8	p ₁ VIII	3513.06	1	28457.1	—
3104.76	1	32199.3	—	3530.80	1	28274.1	d ₁ VIII
3108.57	(2)	32159.8	p ₂ VIII	3550.98	1	28153.2	II VIII
3142.99	1	31807.6	p ₁ D	3557.40	1	28102.4	I D
3193.09	1	31308.6	p ₁ d ₃	3621.97	1	27601.4	I d ₃
3249.49	3	30765.1	p ₂ D	3637.35	3	27484.7	p ₁ p ₂
3265.79	4	30611.6	p ₁ d ₂	3645.57	8	27422.7	III VIII
3303.26	5	30264.4	p ₂ d ₃	3650.31	4	27394.6	d ₂ VIII
3337.67	15	29952.4	p ₁ d ₁	3706.02	5	26975.5	p ₁ p ₁
3344.71	7	29889.4	p ₃ d ₂	3713.71	6	26919.6	d ₁ D
3381.10	10	29567.7	p ₂ d ₂	3715.03	3	26910.0	p ₂ p ₃

The wave-lengths and intensities are borrowed from the Handbuch of KAYSER according to EXNER and HASCHEK's observations, (spark

λ	I	$\nu_{vac.}$	Combination	λ	I	$\nu_{vac.}$	Combination
3715.67	4	26905.3	I d ₂	4316.07	1	23152.7	IV d ₁
3725.24	3	26836.3	—	4322.69	5	23127.3	VI D
3736.02	5	26758.9	II D	4333.97	15	23057.0	VII D
3780.85	3 _r	26441.6	p ₂ $\overline{p_2}$	4364.85	3	22903.9	II $\overline{p_3}$
3784.95	2	26413.0	IV VIII	4383.62	5	22805.8	—
3808.89	2	26246.9	I d ₁	4430.11	8	22566.5	VII d ₃
3835.29	5	26066.3	p ₃ $\overline{p_1}$	4436.02	1	22536.4	—
3840.92	5	26028.0	III D	4455.99	2	22435.4	II $\overline{p_2}$
3846.16	3	25992.6	$\overline{d_2}$ D	4474.72	(3)	22341.5	—
3849.20	10	25972.1	V VIII	4526.30	5	22086.9	$\overline{d_1}$ $\overline{p_1}$
3855.10	3	25932.3	p ₂ $\overline{p_1}$	4558.66	4	21930.2	VI d ₂
3886.51	15	25722.7	$\overline{d_1}$ d ₂	4559.51	2	21926.0	II $\overline{p_1}$
3910.95	(2)	25562.0	II d ₂	4571.14	(2u)	21870.4	VII d ₂
3916.21	10	25527.7	III d ₃	4575.08	4	21851.4	—
3921.71	10	25491.9	$\overline{d_2}$ d ₃	4605.99	2	21704.8	III $\overline{p_2}$
3988.66	30	25064.0	$\overline{d_1}$ d ₁	4613.57	4	21669.1	$\overline{d_2}$ $\overline{p_2}$
3995.91	5 _r	25018.6	IV D	4647.64	1	21510.3	—
4026.03	4	24831.4	III d ₂	4669.10	3	21411.4	—
4031.86	7	24795.5	$\overline{d_2}$ d ₂	4699.80	(2)	21271.6	VI d ₁
4067.52	8	24578.1	V D	4713.08	(8)	21211.6	VII d ₁
4076.85	2	24521.8	VI VIII	4716.59	(8)	21195.8	III $\overline{p_1}$
4077.51	15	24517.9	IV d ₃	4724.57	(5)	21160.0	$\overline{d_2}$ $\overline{p_1}$
4086.90	20	24461.5	VII VIII	4728.55	(10)	21142.2	—
4141.90	10	24142.6	$\overline{d_2}$ d ₁	4804.22	(7)	20809.2	—
4152.17	10	24077.0	V d ₃	4824.22	(8)	20723.0	V $\overline{p_3}$
4152.97	5	24072.4	—	4830.68	(2)	20695.2	IV $\overline{p_2}$?
4196.74	10	23821.3	IV d ₂	4935.77	(2)	20254.6	V $\overline{p_2}$
4204.23	4 _r	23778.9	I $\overline{p_2}$	4946.60	(5)	20210.3	—
4238.57	10	23586.2	—	4952.21	(4)	20187.4	IV $\overline{p_1}$?
4250.17	6	23521.8	—	4986.99	(6)	20046.6	—
4275.80	4	23380.9	V d ₂	5063.07	(2)	19745.4	V $\overline{p_1}$
4296.21	8 _r	23269.8	I $\overline{p_1}$	5464.57	(5)	18294.6	VI $\overline{p_1}$
4300.59	3	23246.1	—	5482.47	(6)	18255.0	VII $\overline{p_1}$

spectrum), those where the intensities are placed between brackets according to WOLFF, (arc-spectrum).

Table B gives the lines, for which the observed ZEEMAN effect renders a calculation of J and g for initial and final state possible, but which cannot yet be classed in the above-given term system. For some of these lines the result is uncertain, possibly because not yet all the parallel components have been observed.

Table C gives the classified lines.

Leiden. Instituut voor Theoretische Natuurkunde.

Histology. — *"The innervation of the muscle-fibres of the myo-cardium and of the atrioventricular bundle of HIS in the heart of the tortoise (emys and cyclemys). (1st Communication). By Prof. J. BOEKE.*

(Communicated at the meeting of November 29, 1924).

In no organ the innervation problem is of such a fundamental importance both for the physiology and for the pathology of it, as it is in the heart-muscle. In no organ, however, the histological study of the innervation meets with so many difficulties and obstacles as in the heart-muscle.

Although there is still some discussion going on about histological details, the mode of innervation of the cross-striated muscle-fibres, the form and distribution of the motor endplates on them, their connection with the muscle-fibres, are well known. In the heart-muscle it is not even known, whether there are distinct nerve-endings on the muscle-cells, or whether there is simply a plexus of nerve-fibres running between the muscle-fibres without distinct endings being formed. In his splendid and exhaustive treatise on the innervation of the heart¹⁾ published in 1908 MOLLARD draws the conclusion "qu'à l'heure actuelle on ne connaît pas encore d'une façon certaine et définitive le mode de terminaison des nerfs dans le myocarde. L'existence de terminaisons motrices établies sur le type de celles les plus simples des muscles striés ordinaires (RENAUT) n'est pas démontrée, non plus que l'existence d'un type spécial de terminaisons différent à la fois de celles des muscles striés qui sont enveloppées dans un sarcolemme, et de celles des muscles lisses (SMIRNOW)" (l. c. p. 185). And even to-day we have to confess that this statement is true.

It has been amply demonstrated for the different classes of the higher vertebrates, how extraordinarily rich in nerve fibres throughout its whole extent the myocardium is, but we are still quite ignorant about the way on which these nerve-fibres are connected with the contractile elements themselves. The majority of the investigators of the subject, and among them especially the authors, who have most thoroughly studied the innervation of the heart-muscle, lay much stress on the fact, that though it is possible to demonstrate a network of very fine nerve fibres surrounding the muscular elements on all sides, no trace of distinct free nerve-endings is to be found. The apparently free endings of the nerve fibres, which show themselves here and there in the preparations, must not be considered as real free nerve-endings, but as the results of a break in the impregnation of the nerve-fibres, a common feature of the

¹⁾ J. MOLLARD. Les nerfs du coeur. Revue générale d'Histologie. Fascicule 9. 306 pages, 79 figures. 1908.

GOLGI-impregnation-method, consequently as a fault of the technique employed in the investigation, and not as a histological reality (HOFMANN). Even a pastmaster of the technique of the staining of the elements of the nervous system as DOGIEL only succeeded in demonstrating in the heart-muscle of the tortoise a very fine network of delicate nerve fibres surrounding the muscle cells, and finished his description of the mode of innervation by saying that in one of his preparations in one and only one place he had got the impression, that there were small knob-like nerve-endings to be seen¹⁾, attached to the surface of the muscle-fibres. The drawing which has to illustrate this point is however not convincing. MICHAILOW²⁾ who in the year 1909 applied the impregnation-method of CAJAL to the heart muscle of the frog, described (just as JACQUES in 1902) distinct ramified nerve-endings on the muscle-cells of the heart, which he took for the endings of the vagus nerve; in addition to these endings he found again the same delicate varicose nerve-plexus surrounding the heart muscle-cells as described by former investigators. According to him however the existence of distinct end-branches, ending in a neurofibrillar ending or endloop on the surface of the muscular elements is undeniable. These endings never enter the muscle-cells themselves. The earlier statements by RANVIER "que les fibrilles nerveuses pénètrent réellement dans les cellules musculaires et qu'elles passent au voisinage de leur noyaux" (1878, 1880) were the results of a defective technique and in consequence an incorrect observation.

Recent investigations have not been able to alter these views. The standpoint of the majority of the investigators to-day is essentially the same as was that of MICHAILOW and DOGIEL in the year 1907 and of GORDON WILSON in the year 1909. Extensive researches, it is true, on the minor details of the innervation of the cardiac muscle-elements, these later years did not bring us. The authors confirmed themselves to the investigation of the distribution of the ganglion cells in the wall of the heart, their connections with the cardiac nerves, their number in the different parts of the heart in different animals, and to the distribution and form of the sensory nerve-endings in the endocard and in the pericard.

This is not to be wondered at, for I don't know of any other tissue, which refuses so obstinately to take a nerve-stain as does the cardiac muscle. For years I have tried again and again to stain the nerve-fibres of the heart-muscle and their endings with the same method, which gave excellent results in the skeletal muscles of the self-same animals, but without success. It was only this year that by means of the neurofibrillar staining-method of BIELSCHOWSKY with a somewhat modified length of

¹⁾ J. DOGIEL. Einige Daten der Anatomie des Frosch- und Schildkrötenherzens. Arch. f. mikrosk. Anatomie. 70. Bd. 1907.

²⁾ S. MICHAILOW. Internat. Monatschrift f. Anat. u. Physiologie. 25. Bd. 1909.

impregnation-time in the Ag NO₃-solution (6 to 6½ days at a temperature of 35° C) I succeeded in getting good preparations with an excellent and elective colouring of the nerve-fibres and their endings in the cardiac muscle of the heart of tortoises and birds, and with tolerable results in the heart of the hedgehog.

In this communication however I will confine myself to the description of the distribution of the nerve-fibres and of the relations between the nerve-fibres and their terminal ramifications and the muscular elements, and the innervation of the muscular elements of the atrioventricular connection (bundle of His), in the heart of the tortoises (*emys europaea*, the European Pond-tortoise, and *cyclemys* sp.), because with those hearts I obtained the best and most complete results.

The form and arrangement of the elements of the cardiac muscle in the chelonian heart, their curious elongated and branched appearance, with the rod-shaped or oval nucleus in the middle, lying in a large quantity of granular sarcoplasm and surrounded by a ring of myofibrillae, which are freely continuous and pass over without interruption from one cell into the adjacent cells, is essentially the same in the heart of the lizard and in that of the cheloniae, and has been described by a number of authors. The arrangement of the muscular fibres, running in bundles, which are closely interwoven in the ventricular portion of the heart, but form a spacious network with thin strands of tissue and wide meshes in the auricular portion, may be considered also as sufficiently known after the descriptions given by BOTAZZI, DOGIEL, KÜLBES and LANGE, LAURENS and ENELLI. This arrangement too has essentially the same features as in the lizard heart, as it was described recently by KRAUSE (1923), and so there is no need to describe these things again here in full.

In preparations stained after the pyridine-silver method of BIELSCHOWSKY, treated afterwards with chloride of gold, and stained lightly with haematoxylin and eosin, the cardiac muscle-fibres show a very delicate but distinct cross-striation of the myofibrillae, and appear to be surrounded by a thin sarcolemma which is connected with the myofibrillae by very delicate but distinct cross-membranes of KRAUSE. The muscle cells are united by their branches into a syncytium without a trace of cellboundaries or junctional lines.

In the wall of the ventricle, as was stated before, the muscle-fibres are arranged in thick bundles, closely interwoven, and running in the sections apparently in all directions without any attempt at regularity. A closer study of serial sections however reveals a distinct regularity of arrangement. I may refer here to the excellent description given by BOTAZZI ¹⁾.

The wall of the ventricle is not a compact structure. Everywhere it

¹⁾ F. BOTAZZI. Ricerche sulla muscolatura cardiale dell' *Emys europaea*. Zeitschr. Allgem. Physiologie VI. 1907. p. 140—194.

consists of a network of closely interwoven bundles of muscle fibres, covered by the endocardium. These bundles however do not form generally a regular cavernous or sponge-like tissue, but they enclose a more or less spacious central cavity, which by means of the ostium atrioventriculare is connected with the cavity of both auricles, and surrounding it a system of additional cavities, so that the blood flows freely between all the bundles of muscle fibres and these bundles even quite near the outer epicardium-covered surface of the ventricular wall are lined by the endocardium.

In the auricles the system of trabeculae is much more spacious, so that the entire auricular wall is much thinner and the bundles are composed of a smaller number of muscle fibres and are lying more widely apart. Here too all the trabeculae are everywhere lined by endocardium, and in the sections the thinner bundles of muscle fibres are often seen to be connected here by thin bands, consisting only of endocardium without a trace of muscle fibres enclosed by it.

In the septum atriorum, the sinus venosus and the wall of the central ventricular cavity we find underneath the endothelial lining of the endocardium in thick bundles the curious layer of smooth muscle cells, described by GASKELL, ROSENZWEIG, BOTAZZI, which abuts on the tunica media of the large veins of the heart (c.f. LAURENS 1913) and which ROSENZWEIG, BOTAZZI and OINUMA held responsible for the oscillations of the tonus of the tortoise heart. LAURENS could find them only in the wall of the auricles, but without doubt they occur also in the wall of the ventricle. BOTAZZI compares them with the fibres of PURKINJE of the mammalian and avian heart.

Now in the first place we find even in the mammalian heart in the connective tissue underlying the endocardium-endothelium in places scattered bundles of non-striped muscle, as first pointed out by SCHWEIGGER-SEIDEL, which have nothing to do with the fibres of PURKINJE and the bundle of HIS-TAWARA. And secondly we find in the septum atriorum and the dorsal wall of the atria bundles of curious branched *cross-striated* muscle-cells (fig. 6), lying as a loose network in the connective tissue, which may be traced in the sections as an uninterrupted bundle into the wall of the ventricle, which with a better right may be compared with the bundle of HIS and the fibres of PURKINJE. But about this point we have more to say later on.

The innervation and the intracardial nervous system of the tortoise (and in general reptilian) heart has been described by a number of authors. For the description of the distribution of the nerves and their course in the heart wall as far as it is visible to the naked eye I can refer the reader to the paper by DOGIEL and ARCHANGELSKY¹⁾ and to

¹⁾ Pflüger's Arch. f. d. Ges. Physiologie 113. Bd. U. 1.
J. DOGIEL. Arch. f. mikrosk. Anatomie 70. Bd.

the paper by LAURENS¹⁾). Nerves can be seen running along the superior venae cavae to the heart. On the right side a branch, the coronary nerve, runs directly from the sinus under the vein to the ventricle, the remainder of this nerve being distributed to the sinus and perhaps a small branch to the right auricle (LAURENS). The nerve on the right side is distributed principally to the left auricle from which can be made out several branches which run to the ventricle along the auriculo-ventricular groove, some ending here and others continuing on to the dorsal surface of the ventricle. Running to the ventral side of the heart are several fine branches which divide and are distributed to the different vessels of the bulbus aortae. The dorsal ligamentum atrioventriculare of DOGIEL contains one or more large nerve branches running to the ventricle, and numerous ganglion cells. According to LAURENS, whose description I follow here, groups of ganglion cells are more numerous in the tortoise heart than in that of the lizard. The largest are two at the opening of the sinus and of the pulmonary veins into the auricles. As was mentioned already, in the dorsal ligament there are numerous groups of ganglia all along the course of the coronary nerve. On the dorsal side of the left auricle, just under the pericardium, there are numerous small groups of ganglia to be seen, some consisting of only two or three cells. The same holds true for the wall of the ventricle. Here too there are a number of small subpericardial groups of ganglia, near the apex as well as near the coronary groove. At the beginning of the septum atriorum, on the right side, there is a collection of ganglia and also several small groups of nerve cells along this side of the septum. In the connective tissue of the auriculo-ventricular groove, particularly on the left side and in the proximity of the bulbus, the ganglionic masses are very numerous, though small, consisting of from two to five cells (LAURENS). In BIELSCHOWSKY-preparations the plexus around the ganglion cells appears to be very rich. By that reason it is impossible to trace distinct nerve-fibres to separate ganglion cells, or from them to the separate muscle fibres. Experimental work (cutting of the vagus and sympathetic nerve, etc.) may perhaps throw some light on this question too.

For the question of the finer details of the innervation the distribution and the course of the nerve-fibres as seen in the sections under the microscope is of more importance.

As it was described already years ago by GERLACH, the nerve-fibres of the heartwall form everywhere a network, a plexus. Large nerves do not enter into the heart-muscle, except the larger nerve-branches, described above, which originate from the groups of ganglia in the septum atriorum and the neighbourhood of the large veins of the heart and pass from here to the wall of the auricles and of the ventricle.

The ramifications of these nerves form a dense nerve-plexus under the endocardium (the "groundplexus" of GERLACH and HOFFMANN).

¹⁾ Anat. Record. Vol. VII.

This groundplexus is very rich, and consists of coarser nerve-strands in the wall of the ventricle, and of very fine nerve-threads in the trabeculae of the auricles. From this fundamental sub-endocardial plexus arise numerous fine fibres, which terminate at different levels of the endocardium in more or less complicated sensory nerve-endings. The heart is, just as in the higher vertebrates, supplied as abundantly with afferent nerves as is the most sensitive skinsurface.

But in this groundplexus are running also the efferent nerve-fibres, and indeed, when we study the plexus there where it lies under the endocardium of the trabeculae carneae of the auricles, or of the finer muscle-bundles of the ventricular wall, we get the impression that this nerve-plexus, whose branches, consisting of very fine nerve-threads, are seen running in different directions even on the smallest trabecles of the atria, is chiefly of an efferent nature, as will be discussed below.

There is no part of the muscle-bundles to be found, where this subendocardial nerve-plexus is absent. We find it even on the smallest muscle-columns of the auricles and of the ventricle, and even there, where as was described before, two of the trabeculae carneae of the auricles are connected by a strand, consisting only of endocardial tissue, without a single muscle-fibre in it, we often see a very delicate neurofibrillar strand running inside this endocardial thread and connecting the groundplexus of one trabecle with that of the other.

Whether the branches of this nerve-plexus form closed meshes or not, is difficult to state. Everywhere we see the neurofibrillar strands branching and ramifying on the surface of the muscle-columns, but where in the sections in most cases only fragments of the plexus are to be seen, it is difficult to state, whether branches of the neurofibrillar strands, once parted, come together again to unite into one single branch, or simply run across each other in different directions.

In a general way the drawing of the groundplexus around the trabecles of the auricle of the frogs heart, given by HOFMANN ¹⁾, furnishes also a good picture of the features of this plexus in the heart of the tortoise.

From this fundamental subendocardial plexus we see everywhere fine neurofibrillar strands passing into the muscle-columns. They are running here at first between the muscle-cells, as it is to be concluded from the study of cross-sections of the muscle-columns and from the fact, that in longitudinal sections of a muscle-bundle these nerve-fibres are often seen running at right angles to the long axis of the muscle-fibres, winding their way across two or three muscle-fibres. But when we follow these neurofibrillar strands during their course, we see them curve round very soon

¹⁾ F. B. HOFMANN. Das intracardiale Nervensystem des Frosches. Arch. f. Anat. und Physiol. Anat. Abtheilung 1902, Tafel IV.

F. B. HOFMANN. Histologische Untersuchungen über die Innervation der glatten und ihr verwandten Muskulatur der Wirbeltiere und Mollusken. Arch. f. mikrosk. Anatomie, 70. Bd. 1907.

and run on in the direction of the muscle-fibres. These nerve-fibres which are running parallel to the long axis of the muscular elements are nearly without exception varicose, the varicosities often following each other so regularly in the course of the nerve-fibre as to suggest the comparison with a string of beads.

Everywhere we see these delicate varicose threads running through the muscle-bundles, and in the straight long muscle-bundles we often meet in the sections through the wall of the ventricle, they are often to be followed a long way passing a number of muscle-nuclei. In the groups of muscle-fibres fixed in a contracted state they often appear as meander-like winded threads. Especially in the sections through the wall of the auricles the short thin muscle-columns running freely through the atrial cavity and often being composed of only two or three muscle-fibres, present even in thin sections (15—25 μ) a very clear picture of the groundplexus underneath the endothelial lining and of the nerve-ramifications between the muscle-fibres inside.

Whether these different intermuscular branches are still running exclusively between the muscle-fibres or are lying already imbedded in the protoplasm of the fibres themselves is not easily to determine in each case given. But it is certain, that *from these intermuscular nerve-threads branch off very fine varicose nerve-fibres, which pass into the muscle-cells and lie imbedded in the protoplasm itself.* They are extremely delicate, beset with small *irregular* varicosities, and either may be followed through two or even three muscle-cell territories, or end abruptly within the sarcoplasm with a small endnet or endloop or a series of terminal varicosities appearing as very small ring-like expansions of the neurofibrillar structure. In fig. 1 several of these nerve-endings are drawn from a section through the auricles, in fig. 2 a cross-section through part of a muscle-column of the ventricle shows the outline of some of the muscle-fibres, the one in the middle showing the nucleus and lying close to it imbedded in the sarcoplasm a small neurofibrillar ring at the end of an extremely delicate neurofibrille coming from the outer part of the muscle-fibre.

The clearness and distinctness of the impregnation of the neurofibrillae enabled us to ascertain with the utmost surety, that we had before us real endings of the neurofibrillar structure and not varicosities cut off by the microtome-knife. And indeed, we find those endings everywhere in the preparations, and where they are lying in the middle part of the sections, so that they are covered above and below by a layer of tissue, we may quite easily ascertain, whether they are real endings with rounded endloops or simply a place, where the neurofibrillar strand was cut across. Especially in the thin muscle-bundles of the auricles the intraprotoplasmic neurofibrillae with their varicosities and small endnets or endrings were distinctly visible as such. In many cases the intraprotoplasmic position of these neurofibrillar endings is made clear by their lying in exactly the same niveau as the nucleus in the clear nearly

uncoloured protoplasm, which surrounds the nucleus especially at both poles and which is surrounded by a ring of cross-striated myofibrillae. Inside this ring we often find the endloops of the neurofibrillar strands. This appears evident in those cases, as are drawn as exactly as possible in figg. 2, 4 and 5, where the neurofibrillar endring is lying quite close to the nucleus, sometimes even enclosing the top of the elongated nucleus. In this regard fig. 5 seems important; here a reticulated varicosity of the neurofibrillar strand was lying so close to the nucleus of a muscle-fibre, that it is making an indentation in the nucleus, and thus was lying in a cavity so deep, that at first sight it made the impression as if two nuclei were present, the neurofibrille passing between them. Only by focussing very carefully it became clear, that there was only one nucleus present, and that the neurofibrillar ring was lying close against it. Another endring too is shown in the same figure, lying also close against the side of the same nucleus. In cross-sections through the muscle-columns we see that the nerve-fibres, which run in the direction of the long axis of the muscle-fibres, are lying as a rule in the peripheral layer of the sarcoplasm, and one is tempted to ask, whether the principal intramuscular fibres, which were described as running with many curves apparently between the muscle-fibres, are in reality all of them lying between the muscle-fibres, and whether they are not, at least a number of them, already enclosed in the protoplasm of the muscle-fibres themselves.

The delicate varicose neurofibrillar threads furnished with endrings and endloops, whose intraprotoplasmic position could be made out convincingly, showed here and there a distinct connection with the sarcoplasm, an extremely delicate network, at one side continuous with the neurofibrillar structure, at the other side apparently passing into the cross-striation of the myofibrillae, at least with meshes, which in size correspond with the width of the cross-striation. In conformity with what is known about the motor-endplates on the cross-striated muscle-fibres of the voluntary muscles and their protoplasmic connection with the myofibrillae, it would be possible to identify this network with the *periterminal network* of the striped muscle-fibres.

In short, we find everywhere in the cardiac muscle an innervation of the individual contractile elements, consisting not only in a pericellular network, which encloses the muscle-fibre without definite nerve-endings (HOFMANN), but in a form, which on one hand reminds us of the mode of innervation of the non-striped muscle-fibres, as it was described for example in the musculus ciliaris of the human eye ¹⁾, on the other hand exhibits some features which remind us of the structure of the motor endplates on the striped muscle-fibre.

And finally, our observations seem to confirm to a certain degree the old statement by RANVIER, which was cited in the beginning of this

¹⁾ J. BOEKE. These Proceedings 17, p. 1982 and 18, p. 2.

paper, "que les fibrilles nerveuses pénètrent réellement dans les cellules musculaires et qu'elles passent au voisinage de leurs noyaux".

Auriculo-ventricular bundle (bundle of HIS). According to DOGIEL and ARCHANGELSKY (for the tortoise) and IMSCHANITZKY (for the lizard) the different parts of the reptile heart are not connected muscularly, the connection between the auricles and ventricle being effected solely by means of a large nerve bundle which runs in a band of connective tissue on the dorsal side of the heart, the "ligamentum atrioventriculare" of DOGIEL (1906).

It seems however, that the contradictory statement by a number of authors (f. ex. BOTAZZI, ROSENZWEIG, KÜLBS and LANGE, LAURENS) is convincing, and the existence of a funnel-shaped muscular connection between the auricular wall and the ventricle seems to be demonstrated beyond doubt.

But, as I mentioned before, it seems to me that two phenomena, which have nothing to do with each other, are often mixed up together in this respect. By BOTAZZI, in his paper cited on a previous page, much importance for the co-ordination of the heart is ascribed to the subendocardial layer of smooth muscle-cells described by him and by ROSENZWEIG and which forms a continuation of the tunica media of the large veins of the heart into the auricles and the ventricle ¹⁾, and when I understood rightly the short description, insufficiently illustrated, of KÜLBS and LANGE, these authors have the same elements in view, when they describe the voluminous funnelshaped muscular connection between the auricles and the ventricle. Now, as is pointed out by LAURENS, there exists without doubt a smaller funnelshaped connection between the auricles and the ventricle, consisting of *cross-striated muscle-fibres*, with a distinct but fine striation and large round or slightly oval nuclei, and of a brighter appearance than the ventricular fibres. This is the real bundle of HIS. It is thickest at the dorsal side of the auricles in the hind part of the septum atriorum, where we find the ligamentum atrioventriculare of DOGIEL, which forms a pathway for nerves and blood vessels and groups of ganglion cells between the sinus and the ventricle. This muscular connection, the real bundle of HIS, is composed of cross-striated muscle cells (c. f. LAURENS), which are more loosely arranged in the connective tissue than the muscular elements of both auricles and ventricle, and therefore present a more branched and netlike appearance (fig. 6), herein exhibiting the same features, which are so characteristic for the fibres of PURKINJE in the avian and mammalian heart. At one side this muscular bundle is connected with the auricular musculature, at the other side with the inner wall of the ventricle.

The striated muscular elements of this bundle receive an individual

¹⁾ According to LAURENS they occur only in the auricles, but they are distinctly visible also in the wall of the ventricle enclosing the central ventricular cavity.

J. BOEKE: "The innervation of the muscle-fibres of the myocardium and of the atrioventricular bundle of His in the heart of the tortoise (*emys* and *cyclemys*)".

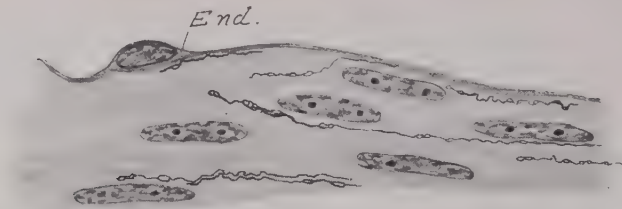


Fig. 1. Nerve-endings in a longitudinal section through a muscle-trabecula of the auricle of the heart of *emys europaea*.
End. = Endocardium.

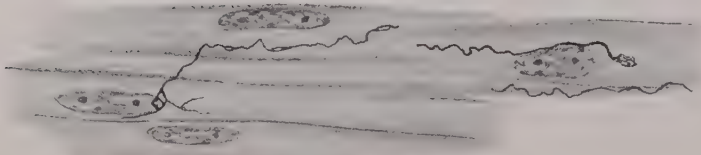


Fig. 2. The same from the wall of the ventricle.



Fig. 3. From a cross section through a muscle-column of the auricle, showing the nucleus of one of the fibres, and besides it an intraprotoplasmic neurofibrillar ending (foreshortened).

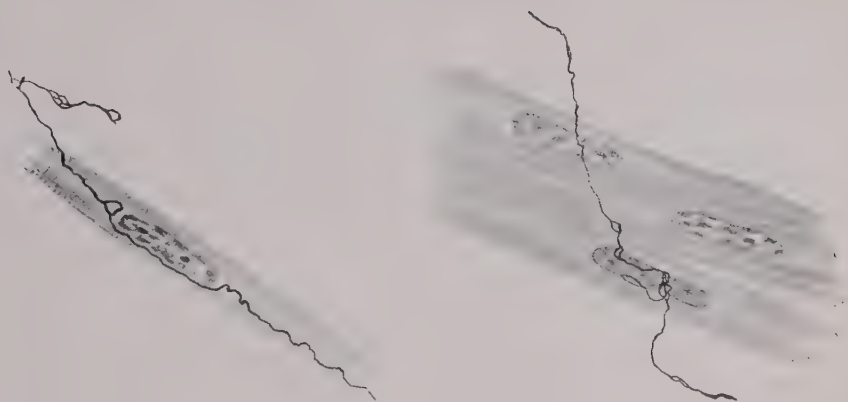


Fig. 4 and 5. Heartmuscle-fibres from the wall of the auricle of the tortoise heart with neurofibrillar nets lying close to the nucleus.

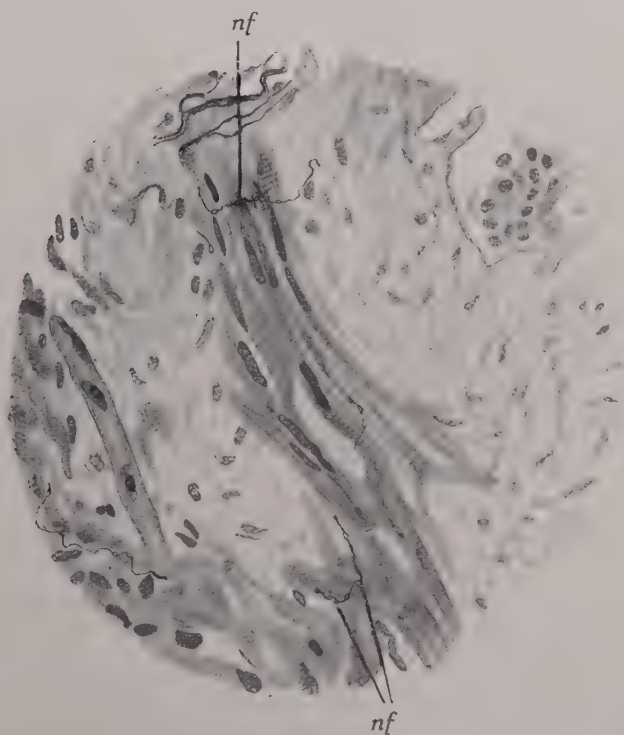


Fig. 6. Longitudinal section through the connective tissue of the septum coronarium with loosely connected muscle-fibres of the atrioventricular bundle of His, and on these fibres small terminal nerve-fibres (*nf.*).

efferent innervation in the same way as it was described in the foregoing pages for the muscle-cells of the auricles and the ventricle. Not only we find, running along it, the large nerve-bundle described above, and in connection with this numerous groups of ganglion cells, but the muscle-elements themselves possess an individual innervation (fig. 6), intraprotoplasmic nerve-fibres with neuro-fibrillar endrings and varicosities (fig. 6 *nf*), and a perimuscular nerve-plexus, which is certainly not less developed than around and in the muscle-columns of the auricles and the ventricle. Every individual muscle-fibre of the bundle of His is innervated.

In conclusion we may remark, that also in the subendocardial layer of smooth muscle-cells of BOTAZZI and ROSENZWEIG referred to above, a plexus of very delicate nerve-fibres with varicosities and small endnets could be traced throughout the whole layer. The strands of this plexus appeared to be in connection with the perimuscular nerve-plexus of the muscle-bundles lying underneath and seemed in many cases to be the direct continuation of this plexus.

Utrecht, November 1924.

Chemistry. — "*Equilibria in systems, in which phases, separated by a semi-permeable membrane*". III. By F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of November 29, 1924).

Ternary systems with solid phases.

We shall consider the membrane-diffusion-pressure and the osmotic attraction in ternary systems, in which one or more solid phases occur. For this reason we take at constant T and P the ternary systems $W-X-Y$ in which W is the diffusing substance; in order to simplify the discussions, we shall assume that W is water; yet the deductions remain valid generally. For the sake of abbreviation we shall represent the membrane-diffusion-pressure of the water by $M.D.P.W.$ and the osmotic water-attraction by $O.W.A.$

1. $M.D.P.W.$ and $O.W.A.$ when one of the components X or Y occurs as solid phase.

In the figs 1—3 the saturation-curve of the component Y is represented by curve wv ; according to a known property, each straight line going through point Y is allowed to intersect this curve in one point only. We shall call w the hydrous and v the anhydrous terminating-point of this curve. Proceeding along this curve starting from w towards v , we arrive firstly in a , then in b etc.; we shall say that a is situated closer to w than b , b closer to w than c , etc.

Going from w to v , we arrive, therefore, always further away from the hydrous terminating-point w , but hence the conclusion may not yet be drawn that the percentage of water of a liquid decreases always, the more it is situated further from w . This is really the case in the figs 1 and 3, in which all lines, parallel to the side XY intersect curve wv in one point only. In fig. 2 this is not the case, we imagine in this figure wc and the tangent in b parallel to XY . Starting from w the percentage of water of the liquid then firstly increases till in b , where it is a maximum; starting from b it decreases; in c the percentage of water is equal again to that in w .

We now shall deduce:

the more a liquid is situated further from the hydrous terminating-point of a saturation-curve, the greater the $O.W.A.$ of a liquid of this saturation-curve is.

The osmotic attraction of water must, therefore, increase along curve wv in the direction of the little arrows on this curve. In figs 1 and 3 the $O.W.A.$ of liquid a is, therefore, larger than that of w , that of b

larger than that of a , etc.; consequently the O.W.A. increases, the more the percentage of water of the liquids becomes smaller. Otherwise it is, however, on part wc of curve wv in fig. 2. Although w and c have the same percentage of water, the O.W.A. of liquid c is larger than that of w and although b has a greater percentage of water than a or w , the O.W.A. of liquid b is greater than that of a and w .

We may deduce in different ways the property mentioned above.

It is clear that the O.W.A. of a saturated liquid a or b is defined by the isotonic curve of the water, which passes through the point a or b . Consequently we have to show that the point of intersection of curve wv with an isotonic curve is situated further away from the point w , the greater the O.W.A. of that isotonic curve is.

In figs 1—3 the isotonic curves are represented by dotted curves. As the O.W.A. of the liquids of those curves increases the more those curves are situated further away from the point W , the little arrows on the sides WX and WY indicate, therefore, the direction of increasing O.W.A. As we have formerly shown, those curves cannot intersect one another. Firstly we shall show:

an isotonic curve and the saturation-curve of a component can intersect one another in one point only, they never can touch one another.

In ternary systems viz. is valid the rule: two saturation-curves, each of a component, can intersect one another in one point only, they never can touch one another. When we now consider the isotonic W -curve like the saturation-curve of the component W in a definite imaginary state (W_1), then at once the above-mentioned property follows from this.

This may be deduced in the following way also. We imagine above triangle WXY of figs 1—3 the ζ surface $W'X'Y'$ belonging to the assumed T and P . On the axis YY' we take a point Y'' in such a way that YY'' represents the ζ of the solid component Y ; on the axis WW' we take a point W'' in such a way that $WW'' = \zeta(w_1)$. The projection of the tangent-curve of a cone, touching the ζ surface and having the top in Y'' , is then the saturation-curve wv . The isotonic W -curve, the O.W.A. of which is defined by WW'' , is the projection of the tangent-curve of a cone, touching the ζ surface and having the top in W'' .

The point of intersection of those two curves is defined, therefore, by the point of contact of the ζ -surface with a tangent-plane, which passes through the line $W''Y''$. It is evident now that, when the ζ -surface is downward convex in each point, there may be one similar tangent-plane only, consequently also one point of intersection only.

We now take an isotonic W -curve of such low O.W.A. that it doesn't intersect the saturation-curve wv ; on increase of the O.W.A. it moves away from the point W till in the stand wm it meets for the first time curve wv . Of all liquids of curve wv , w has, therefore, the lowest O.W.A.

We now take the two liquids a and b . As those liquids, like we have deduced above, cannot be situated on a same isotonic curve, they are situated, therefore, on the two different curves an and bo , and as those curves can not intersect one another, they are situated with respect to another as is drawn in figs 1—3. Consequently the *O.W.A.* in b is greater than that in a . We find, therefore, the above-mentioned property:

the *O.W.A.* of a liquid of a saturation-curve is greater, the further this liquid is situated from the hydrous terminating-point of the curve.

Of course the reserve is the case with the *M.D.P.W.*

We may deduce this property also in the following way. The saturation-curve wv of the component Y is defined by:

$$\zeta - x \frac{\partial \zeta}{\partial x} + (1-y) \frac{\partial \zeta}{\partial y} = \zeta y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which ζy represents the ζ of the solid component Y . The *O.W.A.* and the *M.D.P.W.* depend on

$$\varphi = \zeta - x \frac{\partial \zeta}{\partial x} - y \frac{\partial \zeta}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The *O.W.A.* increases at decreasing value of φ . From (1) and (2) follows at constant T and P :

$$\begin{aligned} [-xr + (1-y)s] dx + [-xs + (1-y)t] dy &= 0 \quad . \quad . \quad . \quad (3) \\ d\varphi &= -(xr + ys) dx - (xs + yt) dy; \end{aligned}$$

with the aid of (3) the latter equation passes into:

$$d\varphi = -s dx - t dy \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We now may show that in no point whatever of curve wv $d\varphi$ can become zero. In accordance with (4) should be necessary for this

$$s dx + t dy = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

while dx and dy satisfy (3) also. This can only be the case, when

$$rt - s^2 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

consequently when the liquid under consideration is situated on the spinodal line of a region of dimixtion; we shall refer to this at the discussion of the dimixtion.

The *O.W.A.* changes, therefore, starting from w towards v always in the same direction, without showing a maximum or minimum. When we know in one point only of curve wv the direction, in which the *O.W.A.* increases or decreases, then, therefore, this direction is known for the whole curve also. For this reason we choose a point in the vicinity of point w . Then x becomes infinitely small in (3) and (4), while xr approaches RT . We now find:

$$d\varphi = -\frac{RT}{1-y} \cdot dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

so that $d\eta$ is negative in the vicinity of the point w . Consequently η decreases, starting from w , so that the O.W.A. increases along curve wv starting from w .

We shall consider now the saturation-curve and an isotonic curve in the vicinity of their point of intersection, f.i. curves wv and nn' with

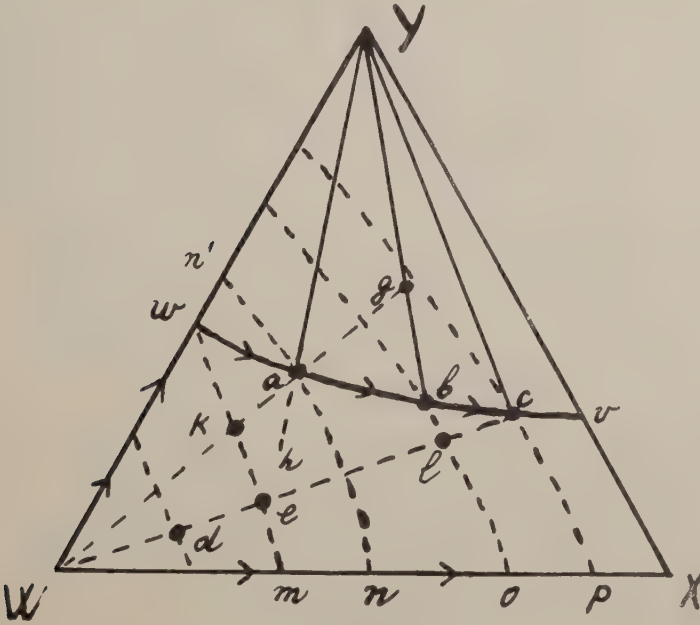


Fig. 1.

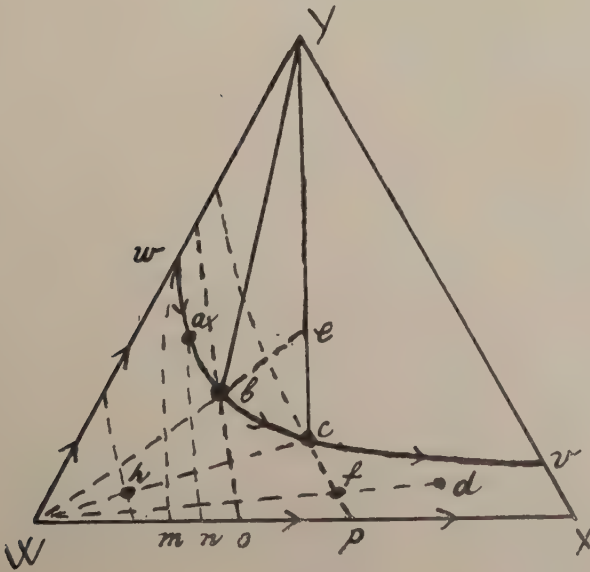


Fig. 2.

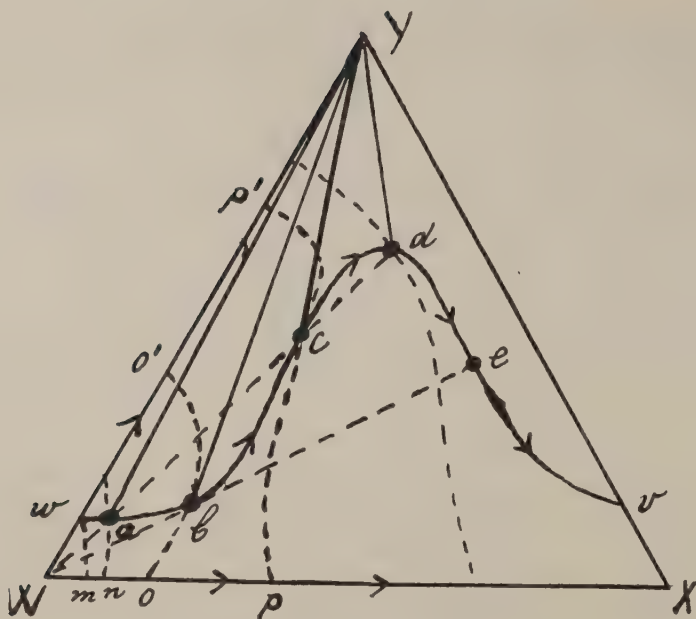


Fig. 3.

their point of intersection a in fig. 1. We shall call the angle WaY and its opposite angle hag the conjugation-angle of both curves, the angles Wah and Yag the supplement-angle.

Formerly¹⁾ we have deduced some rules for the intersection of two saturation-curves; when we replace in those rules one of the saturation-curves by an isotonic curve, then they become:

a saturation-curve and an isotonic curve are situated in the vicinity of their point of intersection either both within the conjugation-angle or both within the supplement-angle.

when the saturation-curve touches one of the sides of that angle, then the isotonic curve touches the other side.

If we imagine in the points a , b and c of figs. 1 and 2 the conjugation angle to be drawn, then we see that in each of those points the saturation-curve is situated within the conjugation-angle, consequently the isotonic curve must also be situated within that angle; the same is the case in the points a and d of fig. 3. In point c of this figure the saturation-curve is situated within the supplement-angle, consequently the isotonic curve pcp' must also be situated within that angle. In point b of fig. 3 the saturation-curve touches the side Wbe ; the isotonic curve obo' must, therefore, touch bY in b .

When L_a viz. the saturated liquid a (figs 1—3) takes water, then it proceeds along the line aW starting from a in the direction towards

¹⁾ F. A. H. SCHREINEMAKERS. Die heterogenen Gleichgewichte von H. W. BAKHUIS ROOZEBOOM III¹ 73, 268.

In-, mono-, and plurivariant equilibria XXV (These Proceedings 27, p. 57).

W ; it gets unsaturated and its $O.W.A.$ becomes smaller. The same is true for all saturated solutions in figs 1 and 2.

When L_c (fig. 3) takes water, then the liquid proceeds, firstly with separation and afterwards again with solution of solid Y , along curve $c b a$ and afterwards along the line $a W$. Also with this it appears again that on addition of water the $O.W.A.$ becomes smaller.

When water is removed from L_a (fig. 1), so that the complex comes in g , then the liquid proceeds along curve $a b$; consequently its $O.W.A.$ increases.

Generally we may say:

at removing (addition) of water from (to) a liquid, its $O.W.A.$ becomes greater (smaller), independent on the fact whether it separates with this solid substance or not.

We now take a liquid L_a (figs. 1—3) and we separate it by a semi-permeable membrane from an arbitrary liquid L_q . We shall say that L_a and L_q are in osmotic relation.

We now distinguish three cases.

1. L_q is situated on the isotonic curve $a n$.

As L_a and L_q have, therefore, the same $O.W.A.$, no water shall diffuse from the one liquid to the other.

2. L_q is situated within the field $a n W w$.

As L_a now has a greater $O.W.A.$ than L_q , water shall diffuse from L_q towards L_a ; this diffusion continues till the $O.W.A.$ of both liquids becomes equal, consequently till they reach the same isotonic curve. As we have seen in the previous communication, it depends on the ratio of the quantities of L_a and L_q which curve this shall be. When we take for L_q f. i. L_d of fig. 1, then L_a may pass into L_k and L_d into L_e . Then the line $e k$ must go through the point, which represents the complex of the original liquids L_a and L_d . In order to represent plainly the conversion, above-mentioned, we shall represent two liquids L_a and L_d which are in osmotic relation by:

$$\left(L_a \begin{array}{c} \downarrow \\ \leftarrow \\ \uparrow \end{array} L_d \right)$$

in which the little arrow indicates the direction in which the water diffuses. We now may represent the conversion by:

$$\left(L_a \begin{array}{c} \downarrow \\ \leftarrow \\ \uparrow \end{array} L_d \right) \longrightarrow \left(L_k \begin{array}{c} \downarrow \\ \longleftrightarrow \\ \uparrow \end{array} L_e \right)$$

in which the double arrow indicates that the liquids L_k and L_e are in osmotic equilibrium.

3. L_q is situated within the field $a n X v$.

As L_a now has a smaller $O.W.A.$ than L_q , water shall diffuse from L_a to L_q . When we take for L_q f. i. L_c of fig. 1, then the conversion becomes:



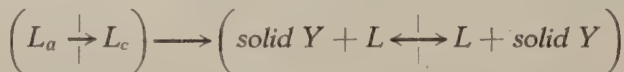
This means, therefore, that L_a gives water to L_c by which L_a passes into solid $Y + L_b$ and L_c into L_l . The line gl will have to go now through the point, which indicates the complex of the original liquids L_a and L_c .

The result of the membrane-diffusion between the two liquids L_a and L_q is, therefore, different, according to the point L_q being situated, on-, at the left- or at the right of the isotonic curve $a n$.

We will yet indicate some conversions, which the reader may easily deduce from the figures. We find f. i. from fig. 2:

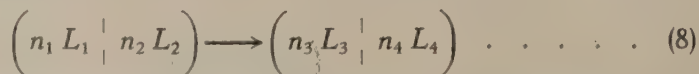


From fig. 3 is found a. o.:



in which L represents a saturated liquid between a and c . Consequently the result of the membrane-diffusion is here, that on both sides of the membrane the solid substance Y is separated and that the liquid on both sides of the membrane gets the same composition. The liquids L_b and L_c , L_w and L_c etc. should give the same result.

We bring n_1 quantities of L_1 in osmotic relation with n_2 quantities of L_2 ; we assume that those n_1 quantities of L_1 pass into n_3 quantities of L_3 and those n_2 quantities of L_2 into n_4 quantities of L_4 . We may write this conversion:



in which the arrows which indicate the direction of the diffusion, are omitted. When we represent the composition of $L_1 L_2$ etc. by $x_1 y_1, x_2 y_2$, etc., then are valid the equations:

$$n_1 x_1 = n_3 x_3 \quad n_2 x_2 = n_4 x_4 \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$n_1 y_1 = n_3 y_3 \quad n_2 y_2 = n_4 y_4 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$n_1 + n_2 = n_3 + n_4 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The two equations (9) express that the quantity of the substance X rests unchanged as well at the one side as at the other side of the membrane, the two equations (10) express the same for the substance Y .

Equation (11) indicates, in connection with the equations (9) and (10) that the total quantity of water does not change with the diffusion. Consequently we have five equations for the definition of the six variables in the second part of (8). We find a sixth equation by the expression that L_3 and L_4 are in osmotic equilibrium, so that they are situated on the same osmotic curve.

Let us take the special case that L_1 and L_2 are situated with point W on a straight line, f.i. $L_1 = \text{liquid } d$ and $L_2 = \text{liquid } l$ from fig. 1. On both sides of the membrane now a same liquid arises, f.i. the liquid e (fig. 1) which we shall represent by L . Inversion (8) now passes into:

$$\left(n_1 L_1 \mid n_2 L_2 \right) \longrightarrow \left(n_3 L \mid n_4 L \right).$$

When we give to L the composition xy , then (9)–(11) pass into:

$$\begin{aligned} n_1 x_1 &= n_3 x & n_2 x_2 &= n_4 x \\ n_1 y_1 &= n_3 y & n_2 y_2 &= n_4 y \\ n_1 + n_2 &= n_3 + n_4. \end{aligned}$$

Herein is, as $L_1 L_2$ and W are situated on a straight line:

$$x_1 : y_1 = x_2 : y_2.$$

The composition xy of liquid L and the quantities n_3 and n_4 of this liquid, which is on both sides of the membrane, are, therefore, defined. We find:

$$\begin{aligned} n_3 &= \frac{n_1 x_1 (n_1 + n_2)}{n_1 x_1 + n_2 n_2} = \frac{n_1 y_1 (n_1 + n_2)}{n_1 y_1 + n_2 y_2} \\ n_4 &= \frac{n_2 x_2 (n_1 + n_2)}{n_1 x_1 + n_2 x_2} = \frac{n_2 y_2 (n_1 + n_2)}{n_1 y_1 + n_2 y_2} \end{aligned}$$

and also:

$$\begin{aligned} n_3 : n_4 &= n_1 x_1 : n_2 x_2 = n_1 y_1 : n_2 y_2 \\ x &= \frac{n_1 x_1 + n_2 x_2}{n_1 + n_2} & y &= \frac{n_1 y_1 + n_2 y_2}{n_1 + n_2}. \end{aligned}$$

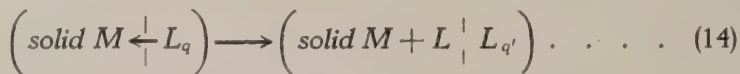
We bring a liquid L_q in osmotic relation with a mixture M of solid substances. When water is diffusing from L_q to the solid substances, then we shall represent it by:

$$\left(\text{solid } M \leftarrow \mid L_q \right); \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

when nothing takes place, then we shall represent it by:

$$\left(\text{solid } M \mid \mid L_q \right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In order to define which of both these cases shall occur, we imagine that an infinitely small quantity of water diffuses from L_q towards the solid substances. A liquid L now arises of absolutely defined composition and changes into a liquid $L_{q'}$ which differs infinitely little from L_q . The inversion is, therefore:



When L has a greater O.W.A. than $L_{q'}$, then still more water shall diffuse to the solid substances; then we have a system (12). When, however, L has a smaller O.W.A. than $L_{q'}$ then the water shall return again to $L_{q'}$; consequently inversion (14) cannot take place, we have, therefore, a system (13).

Consequently it is dependent on the O.W.A. of liquid L whether the solid substances may be able to remove water from the liquid L_q or not. For this reason we will say:

the osmotic water-attraction of a solid substance or of a mixture of solid substances is the O.W.A. of the liquid, which arises when a very small quantity of water is added to this substance.

Consequently when we bring the component Y in osmotic relation with a liquid L , then it depends on the O.W.A. of liquid L_w (figs. 1—3), what will happen. As all liquids of curve $w m$ have the same O.W.A. as L_w and, therefore, also as the solid component Y , we shall call this curve: curve (O.W.A. = Y).

Further we shall say that a liquid, situated in the field $w m W$, is situated closer to W than curve (O.W.A. = Y); of the other liquids we shall say that they are situated further from W than this curve.

We now easily find:

the component Y , in osmotic relation with a liquid, which is situated closer to W than curve (O.W.A. = Y) shall remove water from this liquid;

the component Y , in osmotic relation with a liquid, which is situated further from W than curve (O.W.A. = Y), cannot remove water from this liquid.

Consequently when we bring solid Y in osmotic relation with one of the liquids L_l (fig. 1), L_a or L_f (fig. 2) than nothing takes place. Reversally, when we bring one of these liquids in osmotic relation with L_w , the latter with separation of solid Y , shall lose its water totally or partially.

When we bring solid Y in osmotic relation with liquid L_d (fig. 1) or L_h (fig. 2) then water shall diffuse towards the solid substance; then either solid $Y + L_w$ is formed or an unsaturated solution.

As a special case of the previous considerations follows at once which

shall take place when a binary liquid, containing the components W and X , is brought in osmotic relation with a new solid substance Y .

When this liquid is situated between m and X (figs. 1—3) then nothing takes place; when it is situated between W and m , then the solid substance Y shall remove water from this liquid; then either $Y + L_w$ or an unsaturated solution of Y in water is formed.

Leiden.

Lab. of Inorg. Chemistry.

(To be continued).

Physics. — "*The Brownian Movement of a Thread*". By A. HOUDIJK and Prof. P. ZEEMAN.

(Communicated at the meeting of November 29, 1925).

In an address on "Experimentell nachweisbare, der üblichen Thermodynamik widersprechende Molekularphänomene" delivered at the 84. Versammlung deutscher Naturforscher und Aerzte in 1912¹⁾ VON SMOLUCHOWSKI treats many phenomena that have been known for a long time (Brownian movement, opalescence of gases in the critical state, oscillation of the gas density about a normal middle value) seen from one point of view.

According to the kinetic theory the condition of an isolated system oscillates about a normal state, and can even very far depart from it under definite circumstances, in consequence of which the phenomena in question ensue.

The conditions through which a system in equilibrium passes, correspond to a canonical distribution, and the chance to a definite system defined by the coordinates p and the moments q is:

$$dW = ae^{-\frac{N}{RT}E} dq_1 dq_2 \dots dp_1 dp_2 \dots dp_n$$

in which E is the total energy.

On integration with respect to all the variables except ε (identical with a q) which determines the deviation of the system, the number of systems at a definite moment with an ε lying between ε and $\varepsilon + d\varepsilon$, is:

$$dW = ce^{-\frac{N}{RT}\chi(\varepsilon)} d\varepsilon \dots \dots \dots (1)$$

This well-known equation — BOLTZMANN's exponential law — is characteristic of the kinetic theory. It indicates how far in consequence of the unordered thermal movement a parameter of a system departs from the value corresponding to stable equilibrium.

VON SMOLUCHOWSKI gives several applications, and treats two of them referring to the deformation of a solid body, which differ, therefore, from the known phenomena of variation of the parameter through the molecular motion.

¹⁾ M. v. SMOLUCHOWSKI, Phys. Zeitschr. 13, 1912 (1069).

One of them, the subject of our experimental investigation ¹⁾ is the displacement of the lower end of a thin thread hanging vertically.

Any departure from the state of equilibrium will have the same probability. The factor a in formula (1) is, therefore, constant. Besides — since we are concerned with gravity and with elastic forces — the work done on a displacement from the state of equilibrium, is a quadratic function of the coordinates, so that we, therefore, get a distribution of deviations according to GAUSS's law of errors.

The strict calculation of the case is complicated, for the very reason that besides the work of gravity, also the work required for the bending should be considered.

The order of magnitude of the mean deviation is, however, also obtained when the thread is conceived as a rod, and only gravity is taken into account.

The mean displacement of the end of the thread becomes:

$$\sqrt{\overline{\delta^2}} = \sqrt{\frac{RT}{N} \frac{2}{\pi a^2 \varrho g}}$$

in which a is the radius of the cross-section of the thread, and ϱ the density of the substance used.

This formula may also be derived by another way.

The difficulties met with in the experimental investigation, are among others radiometric phenomena and vibrations of the ground.

In main lines the arrangement was as follows.

The thread attached at one end, was placed in a small tray covered at the front and at the backside by a cover-glass. The free end of the thread was projected on the slit of a falling plate outfit, as is used among others to record electro-cardiograms. A PHILIPS Tungsten arc-lamp with an object glass as condenser serves for the illumination; for the projection is used a microscope with horizontal tube, provided with an apochromatic object glass and a compensation-eyepiece. The whole apparatus stands on a free-stone plate, attached to three free-stone pillars which are mounted on one of the heavy concrete floors of the physical laboratory "Physica". These floors weigh ± 250.000 kg., and rest — clear of the building — on a great number of poles driven in deep. Moreover, the observations were made in the night, between 1.30 and 3.30 a. m., after it had been ascertained that the movement of the floor has a large period only then. With a diaphragm the smallest quantity of light possible for the projection was transmitted.

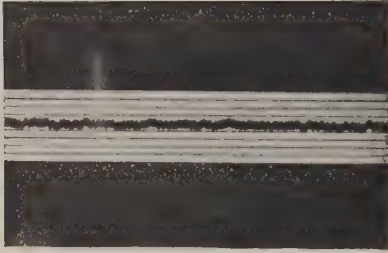
For a long time observations were also made in the hall of the country-

¹⁾ This investigation, in progress for a long time already, is mentioned in:

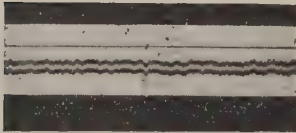
A. EINSTEIN. Untersuchungen über die Theorie der "Brownsche Bewegung". OSTWALD's Klassiker N^o. 199. Anmerkung 15. 1922.

M. v. SMOLUCHOWSKI. Abhandlungen über die Brownsche Bewegung und verwandte Erscheinungen. OSTWALD's Klassiker N^o. 207. Anmerkung 30. 1923.

house of one of us at Huis-ter-Heide, and a few times in the cellar of a house at Zeist. In a nightly observation at Huis-ter-Heide, made by one of us, the deviations of the end of the thread were directly observed, and the magnitude estimated by the eye piece micrometer. Many observations were made in the older physical laboratory, where the apparatus was mounted on a much smaller block of concrete.



Platinum 1 μ .



quartz 2 μ .

Further details and results will be given in the thesis for the doctorate of A. HOUDIJK, to be published later.

In the experiments the photographs are, of course, obtained with the thread vertical.

Wherever the investigation was performed, it was always found that the aspect of the phenomenon was the same.

The subjoined pictures give an idea of the photographs obtained.

The measurements were made with the microphotometer, and visually by enlarging the image.

The results, both of the last-mentioned measurement and those of the above-mentioned direct estimation, are in good agreement with the formula derived above.

The investigation was made with threads of different materials and different cross-sections.

Geology. — "*The tectonics of the central part of the Sierra de los Filabres (South-Spain)*". By H. A. BROUWER and C. P. A. ZEYLMANS VAN EMMICHOVEN.

(Communicated at the meeting of November 29, 1924).

In his outlines of the geology of Spain R. DOUVILLE¹⁾ writes: "Au point de vue tectonique la chaîne bétique est encore inexplorée". The following will be for the main part a short review of results concerning the tectonics of the Sierra de los Filabres²⁾. Till now the admitted stratigraphic division was:

1. Crystalline schists of prae-triassic age.
2. Limestones, shales, slates and phyllites, rauchwackes, gypsum and dolomites, which in analogy with fossil-bearing sediments in the Sierra de Gádor were considered as being triassic.
3. Tertiary and more recent sediments.

We can add to this that also at different places of the Sierra de los Filabres typical triassic fossils (i.a. *Myophoria*) have been found (west of the hamlet La Yedra; south of the Menas-mines; in the Barranco del Pozo de la Nieve and west of the Tetica de Bacares). Whether really all crystalline schists are of prae-triassic age, can here be left out of discussion just as an exact determination of the stratigraphy of the trias — though these facts are of great importance for the tectonics — our first intention being to give only a general idea of the tectonic features. Among the crystalline schists garnetiferous micaschists are predominating; in some horizons they are alternating with many metamorphic igneous rocks (serpentines, amphibolites, gneisses), in other parts they consist only or nearly only of micaceous marbles.

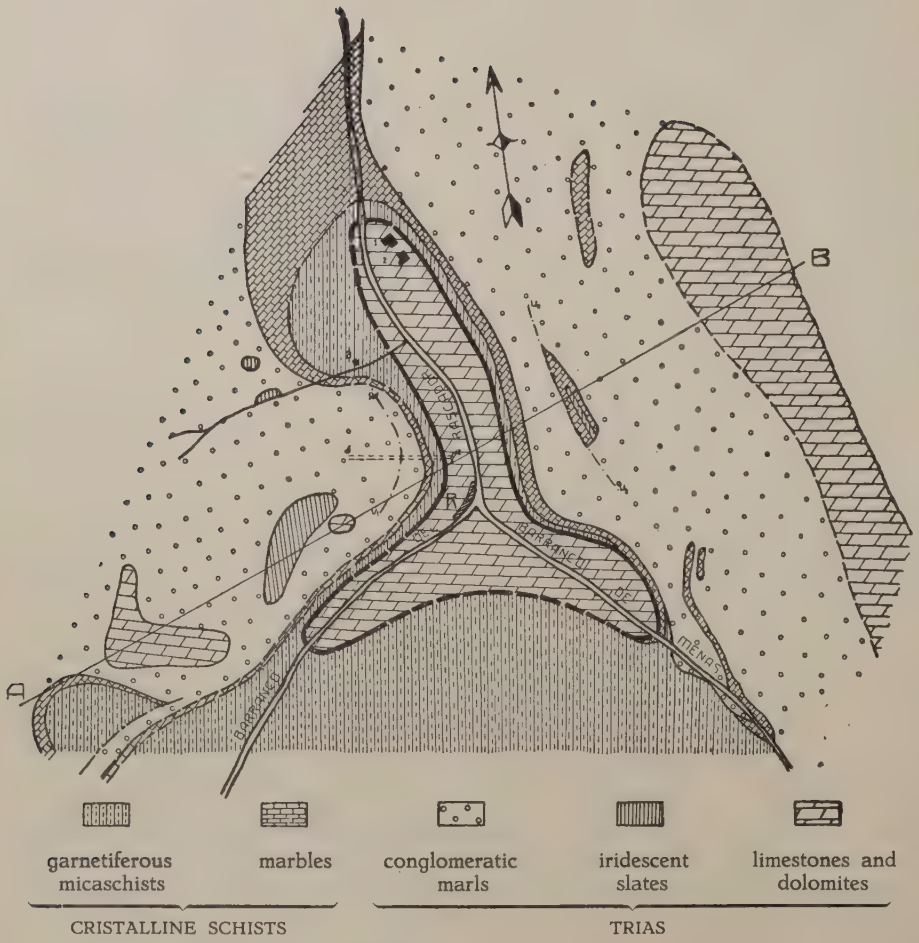
THE POST-TRIASSIC, PRAE-PLIOCENE TECTONICS IN CROSS-SECTION.

The Sierra de los Filabres belongs with its western prolongation, the Sierra Nevada, to the tertiary mountains, which can be followed in W.S.W.-E.N.E. direction from Gibraltar to Alicante. In the northern

1) R. DOUVILLE. La Péninsule Ibérique A. Espagne. Handbuch der Regionalen Geologie. Heft 7.

2) The investigations, of which here a review is given, commenced during an excursion led by H. A. BROUWER and R. W. VAN DER VEEN. These initiated more detailed studies by C. P. A. ZEYLMANS VAN EMMICHOVEN, one of the participants of this excursion.

Fig. 1.



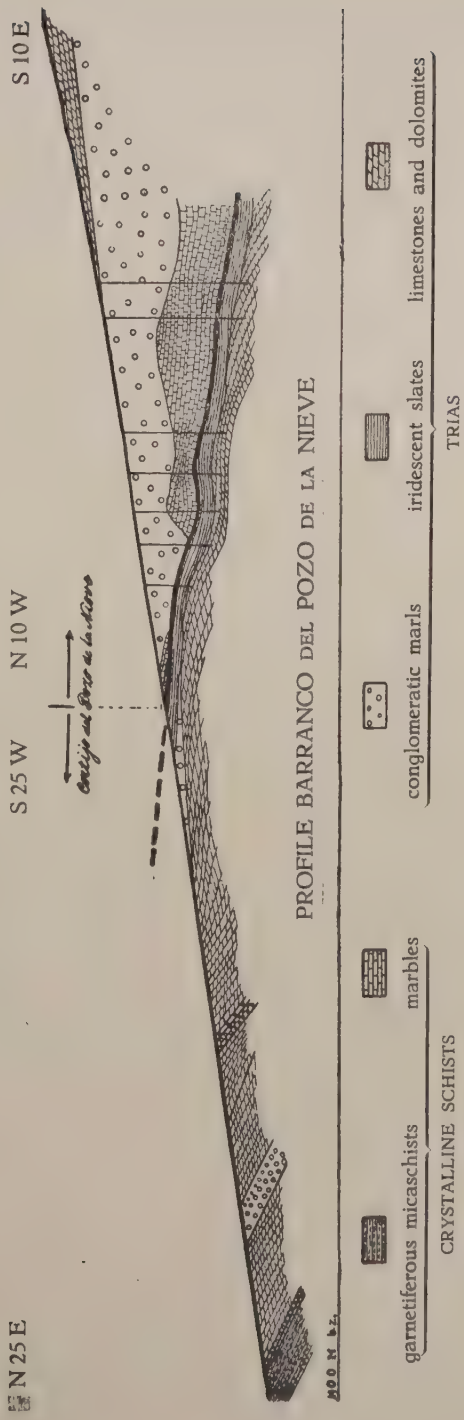
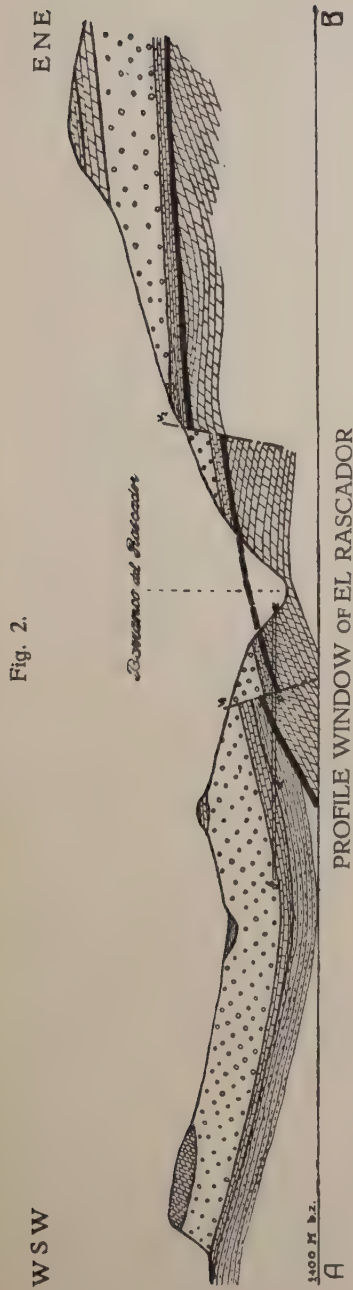
— — — — — outcrop overthrustplane.

- - - - - faults

R El Rascador.

Scale 1 : 15.000.

WINDOW OF EL RASCADOR.



Vertical- and horizontal scale for both fig. 1 : 10.000

Fig. 3.

subbetic mountains DOUVILLE ¹⁾, NICKLES ²⁾ and GENTIL ³⁾ have pointed out the existence of large overthrusts which show movements to the N. and N.N.W. From the following will appear that the structure of the betic mountains in the central part of the Sierra de los Filabres shows characteristics, which likewise point out important horizontal movements in N. and N.W. direction. These facts fit in the general scheme of structure of the Alps of which the betic ranges may be considered to be the direct western continuation.

Of our observations, which prove the existence of large overthrusts in the Sierra de los Filabres, the most important are :

The window ("Fenster") of El Rascador. (Figs. 1 and 2.)

At the confluence of the Barranco del Rascador and the Barranco de Menas the bottom of the valley consists of triassic limestones. As well down-river as up the stream at a distance of about 500 M. from the confluence these triassic limestones dip under cristalline schists which here are for the greater part marbles, with some garnetiferous micaschists.

The triassic-limestones show strong pressure-phenomena near the abnormal contact-plane, while the superposed garnetiferous micaschists have also been strongly squeezed out. The abnormal contact-plane between these two places can also be followed along the slopes of the two barrancos, especially at the east-side. So we find here a window of triassic limestones in the cristalline schists. On top of the latter lie again triassic sediments, consisting of conglomeratic marls, rauchwackes, limestones and dolomites.

Borings S. of the Cortijo del Pozo de la Nieve. (Fig. 3.)

In the upper course of the Barranco del Pozo de la Nieve at a distance of more than 1.5 K.M. east of the window of El Rascador, W. F. C. ENGELBERT VAN BEVERVOORDE, mining-engineer of the Sociedad Minera Cabarga y San Miguel, brought down a series of borings in a line with an almost S.-N. direction.

The results are given in the S. part of profile fig. 3.

¹⁾ R. DOUVILLE. Esquisse géologique des Pré-alpes subbétiques (Partie Centrale). Thèse de doctorat. 1906.

²⁾ R. NICKLES. Sur l'existence des phénomènes de charriage en Espagne dans la zone subbétique. Bull. de la Soc. géol. de France. 4e série, IV, p. 223, 1904.

³⁾ L. GENTIL. Sur l'existence de grandes nappes de recouvrement dans la province de Cadix. Comp. Rend. des Sc. de l'Acad. des Sc. CLXVI, p. 1003, 1917.

Id. Sur l'existence en Andalousie des nappes de recouvrement de la province de Cadix id. CLXVII, p. 83, 1918.

Id. Sur l'origine des nappes de recouvrement de l'Andalousie. id. p. 238. 1918.

Id. Sur l'âge des nappes de recouvrement de l'Andalousie et sur leur raccordement avec les nappes pré-rifaines (Maroc. septentrional) id. p. 373, 1918.

Here we see a 75 M. thick deposit of marbles rather suddenly thinning out to the N., extending under a thick series of conglomeratic triassic marls and rauchwackes, followed by about 20 M. of strata which produced in the boring-samples a clay of a light-blue colour. Thereafter the borings yielded intensively folded triassic limestones. After having perforated a thickness of 80 M. of these limestones the borings were stopped.

These observations were completed by the profile in the near valley.

Going from S. to N. we found :

Conglomeratic trias-marls and rauchwackes.

Crystalline schists, for the greater part consisting of marbles. (Undistinct outcrops).

Triasslates, about 20 M.

Trias-limestones (very thick).

Conglomeratic trias-marls 90 c.M.

Crystalline schists (75 c.M. marble on top of garnetiferous micaschists).

Between this abnormal succession and the one of the window of El Rascador exists this difference that the \pm 20 M. layer of triassic slates in the profile of the borings was not found in the window.

STRIKES AND FRACTURES.

In describing the tectonics in the direction of the folding axes we mention in the first place the complicated structures originated at greater depths during the older tertiary phases of mountain-building, and afterwards uncovered by erosion. Of the newer phases we don't know these complicated structures at greater depths, but only the major relief-features and the movements along fractures. This distinction is not made to make a sharp division between certain periods of intense mountain-building, but because only in this way we are able to give a concise description of the structure of the mountain chain.

Post-triassic, prae-pliocene strikes.

North of a line of E.-W. direction over Macael, which we will describe later as a striking longitudinal fracture-line, we do not find any sediments older than of triassic age. In these triassic sediments the strikes have a S.W.-N.E. direction. South of this line the strikes of the trias are on the whole strongly variable, but in some parts the general direction of strike can be well defined. At both sides of the upper-course of the Barranco del Rascador we find S.W.-N.E. directions, farther to the east the strike is generally E.-W., which still farther eastward in the vicinity of the Calar de Layón, is distorted to a N.W.-S.E. direction. Proceeding to

the E. the strike gradually approaches its former E.-W. direction. For the sake of brevity these tectonics will be called the tertiary tectonics.

The youngest tectonics.

Young-tertiary and quaternary sediments cover unconformably the older ones along the northern border of the mountain range. The trend of these strata is more or less E.-W., the dips amount to an average of about 30° to the north; dips to the south are likewise to be found.

Concerning the horizontal projection of the geanticlinal axis of the present mountain-chain, can be said, that it cannot be stated with certainty where the strongest recent uplift has taken place, because the topography has been influenced by erosion. We can state though, that there are deviations from the general E.-W. trend of the mountain-chain; the horizontal projection of the geanticlinal axis shows a few bending-points and near these bending-points important transversal fractures are found, just as occurs between the Sierra Nevada and the Sierra de Los Filabres on a larger scale.

These *transversal fractures* are from E. to W.:

1. a system of S.S.W.-N.N.E. fractures, S.E. of Macael;
2. the fracture of the Rio de Bacaes, likewise with a S.S.W.-N.N.E. direction;
3. the S.W.-N.E. trending fractures of the Rios del Fargalí and de las Herrerías. It is of importance that the fracture-lines stated sub 2 and 3 are situated at both sides of the E.-W. strikes in the trias, mentioned with the tertiary tectonics, so thus — at any rate this being the case with the eastern ones — in the vicinity of a bending-point of the tertiary strike. The transverse fractures, along which sometimes a horizontal movement is clearly evident, are the expression near the surface of similar movements with horizontal differences of velocity as those, which formed the distortions of the tertiary strikes at greater depths.

Of the *longitudinal fractures*, which are also of much importance for the youngest tectonics, the most obvious is the great E.-W. directed fracture-line, which can be followed in a western direction from Macael till S.W. of Serón. Along this fracture-line the southern part is relatively upheaved, the dip of the fractures plane is steep and often to the south; so we see, that also the youngest tectonics show evidences of reversed faulting in a northern direction.

Thus the transversal as well as the longitudinal fractures show characteristics, from which an analogy between the recent orogenetic movements, and those, by which the tertiary tectonics were formed, is evident.

THE PRAE-TRIASSIC TECTONICS AND THEIR DEFORMATION BY
THE POST-TRIASSIC MOVEMENTS. (Fig. 4).

The prae-triassic tectonics show to the north of the plain of the Guadalquivir, where younger mountain-building movements did not take place, a

distinct more or less N.W.-S.E. strike. These hercynic strikes are cut off under a sharp angle by the important W.S.W.-E.N.E. trending fault, confining the young basin of the Guadalquivir to the north. In the subbetic chains prae-triassic sediments are not uncovered, but in the betic mountains sediments, showing evidences of the hercynic orogenetic forces are again exposed.

It is important, that the same N.W.-S.E. strikes of the prae-triassic sediments are found in the Sierra de los Filabres. In this mountain-chain we see a part of the direct S. E. continuation of the hercynic structures of the Sierra Morena. Still farther south in the Sierra de Alhamilla the same strikes are found to be predominant in the cristalline schists¹⁾.

The subbetic and betic mountains were formed by the younger Alpine movements with a W.S.W.-E.N.E. strike, cutting the direction of the hercynic mountains obliquely.

In the Sierra de los Filabres the influence of the younger movements on the ancient strikes is clearly to be determined. Although the ancient hercynic strike in prae-triassic sediments has mostly disappeared by the new orientating influence of the post-triassic movements, the hercynic strikes can still be followed with the aid of petrographically characteristic strata. If we follow f.i. zone of marbles in the cristalline schists, than we see it extending in a northwestern direction from the Barranco de Sobrino till N.W. of the window of El Rascador. Thereafter the zone bends in a western direction, this even being a W.S.W. one for a short distance west of the Barranco del Fargalí, to take from there to the west, till past Alcóntar, gradually the old hercynic (N.W.-S.E.) strike of the Sierra Morena.

This deviation from the hercynic strike, shown by the zone of marbles, can be brought in direct relation with the bending of the tertiary strike, as this has been mentioned above for the triassic sediments south of the longitudinal fracture-line Macael-Serón.

Bearing in mind that the bending of the tertiary strikes is due to differences in velocity of horizontal movements at greater depth, then these differences of velocity will have altered also the strike of the zones of similar petrographic composition in the cristalline schists.

Assuming an originally N.W.-S.E. directed hercynic strike and a W.N.W.-E.N.E. direction of the strikes during the primary phases of the Alpine mountain-building, deformations of the hercynic and those of the original younger strikes turn out to be caused by the same differences of velocity of the mountain-building movements, which is clearly shown in fig. 4.

We draw therefore the conclusion, that the hercynic mountains have continued from the Sierra Morena in a southeastern direction into the

¹⁾ W. H. HETZEL. Bijdrage tot de geologie van de Sierra Alhamilla. Proefschrift 1923, 's-Gravenhage.

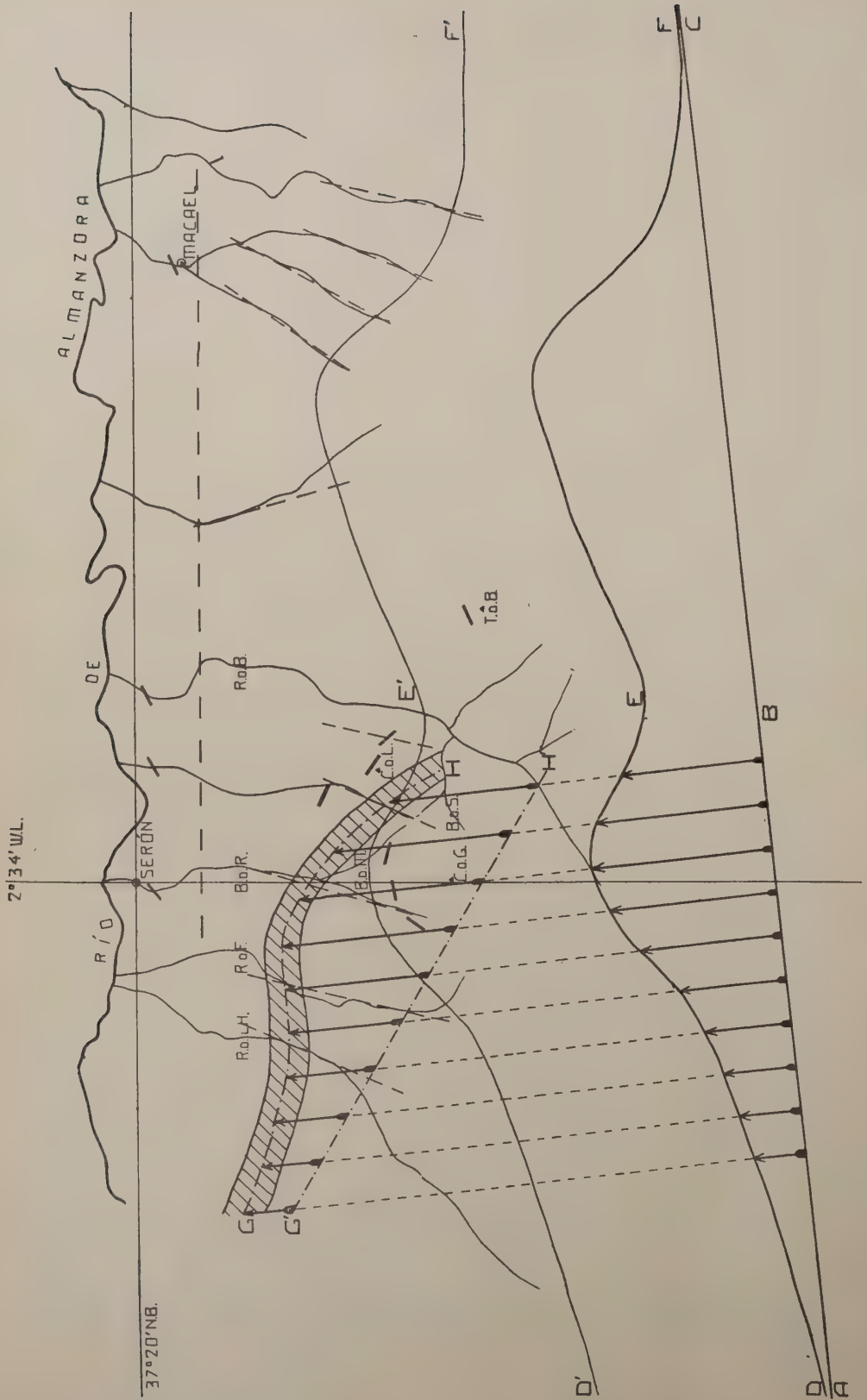


Fig. 4.

Scale 1 : 200,000

EXPLANATION OF FIGURE 4.

The deformation of the prae-triassic tectonics by the post-triassic movements.



zone of marbles in the crystalline schists.

— — — faults.

== tertiary strikes in the trias.

A B C direction of the geanticlinal axis if no differences in velocity of the horizontal movements had taken place.

D E F approximate form of the present geanticlinal axis.

D' E' F' is D E F displaced to the north.

G' H' supposed hercynic direction of the zone of marbles.

R. d. I. H. Río de las Herrerías.

R. d. F. Río del Fargali.

B. d. R. Barranco del Rascador.

B. d. M. Barranco de Menas.

B. d. S. Barranco de Sobrino.

R. d. B. Río de Baces.

C. d. G. Calar de Gallinero 2050 M.

C. d. L. Calar de Layón 1980 M.

T. d. B. Tetica de Baces 2081.40 M.

Mediterranean region. So we see here a great difference between the directions of the younger and the ancient mountains, an analogon of which are the hercynic relict-structures as they were described in Stiria by Heritsch ¹⁾).

¹⁾ FR. HERITSCH. Die Grundlagen der alpinen Tektonik. Berlin. GEBR. BORNTRAEGER 1923, p. 14.

Chemistry. — “*The optically active components of chlorosulphoacetic acid*”. By H. J. BACKER and W. G. BURGERS. (Communicated by Prof. F. M. JAEGER.)

(Communicated at the meeting of January 31, 1925).

The failure of various attempts to resolve racemic chlorosulphoacetic acid $\text{SO}_3\text{H} \cdot \text{CHCl} \cdot \text{CO}_2\text{H}$ by the usual methods suggested some doubt as to the applicability of the theory of asymmetric carbon to simple compounds ¹⁾.

Preliminary observations on its resolution ²⁾ were not confirmed by POPE and READ, who then succeeded in the resolution of another simple compound, namely chloriodomethanesulphonic acid ¹⁾.

Supposing that the failures might be caused by racemisation, we have attempted the resolution by “cold crystallisation” of alkaloidal salts. This method indeed succeeded.

The ammonium or sodium salt of chlorosulphoacetic acid was treated in aqueous solution with two molecules of the acetate or phosphate of strychnine.

The dilution was chosen so, that only a small part of the strychnine salt, about one fifth, crystallised. Decomposed by ammonia at 0°, it gave an active, dextrorotatory ammonium salt.

For purification of the strychnine salt, the same “cold crystallisation” was applied. Accordingly it was decomposed by ammonia and in a suitable dilution again treated with the acetate of strychnine. In this way it could be ascertained when the maximum value of the rotation was attained.

Decomposition of the ammonium salt by sulphuric acid gave the dextrorotatory acid.

The laevo-component was obtained in the same way by means of cinchonine.

The following values of the molecular rotatory power were found :

Wavelength λ (μ)	589	533	494
Chlorosulphoacetic acid [M]	39°	50°	62°
Neutral ammonium salt [M]	20°	26°	34°

The active acids and salts racemise slowly at room temperature. On evaporation on a water-bath the solutions lose their activity completely. In alkaline solution the salts are quickly racemised.

A detailed account of this research will appear in the „Journal of the chemical Society of London”.

Organic Chemical Laboratory of the University.

Groningen, January 1925.

¹⁾ POPE and READ, Journ. chem. Soc. **93**, 794 (1908); **105**, 811 (1914).

²⁾ PORCHER, Bull. Soc. chim. (3) **27**, 438 (1902).

Chemistry. — "*Optical resolution of bromosulphoacetic acid*". By H. J. BACKER and H. W. MOOK. (Communicated by Prof. F. M. JAEGER.)

(Communicated at the meeting of January 31, 1925).

The resolution of chlorosulphoacetic acid, described in the preceding communication, led us to attempt the application of the same method to an analogous asymmetric compound, namely bromosulphoacetic acid, $\text{SO}_3\text{H} \cdot \text{CHBr} \cdot \text{CO}_2\text{H}$.

Sodium bromosulphoacetate gives in the cold crystalline salts with various alkaloids, dissolved in dilute acetic acid. These salts were examined polarimetrically after careful decomposition with ammonia and elimination of the alkaloid.

Strychnine and cinchonidine gave a dextrorotatory ammonium salt, whilst brucine, chinidine and yohimbine led to the laevocompound. Cinchonine gave variable results.

Strychnine and brucine proved to be the most suitable alkaloids for preparing both enantiomorphs in a pure state.

The *dextrorotatory acid* was prepared from 40 m.mol. sodium bromosulphoacetate and 80 m.mol. acetate of strychnine in a dilution of 2100 c.c. After a day one fifth of the total amount had crystallised.

The product was decomposed with ammonia and treated with strychnine in acetic solution in such dilution, that again only a part crystallised. The rotatory power of the ammonium salt had reached its maximum and was not changed by two further crystallisations.

Another method of controlling the optical purity of the strychnine salt consists in washing the finely powdered substance with water. The pure product did not change by this treatment, whilst the rotation of a less pure specimen, examined as ammonium salt, rose to the value of the pure salt.

As even a small excess of a strong base immediately causes racemisation, the d-barium salt was not prepared from the strychnine salt with baryta, but with barium iodide. The dextrorotatory acid, prepared both from the d-barium salt and from the d-ammonium salt with sulphuric acid, had double the rotation of the ammonium salt in the same dilution.

Brucine served for the separation of the *laevorotatory acid*.

As the solubilities of the brucine salts of the two enantiomorphs do not differ much, it appeared desirable to eliminate first a part of the dextro-compound by means of strychnine.

Thus from 20 m.mol. of *r*-bromosulphoacetic acid a fifth part was

separated as strychnine salt, and the mother liquor, decomposed by ammonia, was treated with the acetate of brucine in a dilution of 2 liters. One tenth part of the remaining acid crystallised as brucine salt.

This product, washed twice with 50 c.c. cold water, was pure and showed as ammonium salt the same rotation as the enantiomorph, in opposite direction.

The rotatory power of the l-acid, prepared from this ammonium salt with sulphuric acid, corresponded to that of the d-acid.

The highest constant values, found for the molecular rotation of the two active bromosulphoacetic acids and their neutral ammonium salts, are reproduced in next table :

$\lambda (\mu \mu)$	589	560	533	510	486
acid [M]	31°	37°	42°	47°	56°
salt [M]	16°	18°	21°	24°	27°.

Racemisation of acid and salt takes place slowly at room temperature. A dilute solution of the acid may be boiled for a few minutes without appreciable alteration.

Traces of strong bases greatly accelerate racemisation of the salts.

Experimental details of this research will be published elsewhere.

Organic Chemical Laboratory of the University.

Groningen, January 1925.

Mathematics. — “*Grundzüge einer Theorie der Kurven*”. By Dr. K. Menger. (Communicated by Prof. L. E. J. Brouwer).

(Communicated at the meeting of January 31, 1925).

1. *Der Kurvenbegriff.* Durch keine der älteren Kurvendefinitionen wurde das, was in der Anschauung für die Kurven charakteristisch ist, restlos erfasst. Die Jordanschen “Kurven” (die eindeutigen stetigen Bilder der Strecke) und die irreduziblen Kontinua können bekanntlich ganze Flächenstücke enthalten, — zu den einfachen Kurvenbögen (den topologischen Bildern der Strecke) gehört schon eine so einfache Kurve, wie die Kreislinie, nicht, — und die Cantorsche Definition der ebenen Kurven als nirgends dichte Kontinua ist auf andre Euklidische Räume prinzipiell unübertragbar. Wir bezeichnen, gestützt auf eine allgemeine Dimensionstheorie¹⁾, als Kurven die eindimensionalen Kontinua. Ein Kontinuum K , in dem (durch eine Metrik oder axiomatisch) Umgebungen definiert sind, heisst demgemäss Kurve, wenn zu jedem Punkt von K beliebig kleine Umgebungen mit diskontinuierlichen (d. h. keine Kontinua enthaltenden) Begrenzungen existieren. Die Dimensionstheorie lehrt, dass die Kurvennatur gegenüber topologischen Abbildungen invariant ist, — dass das Intervall des R_1 und mithin jeder einfache Kurvenbogen eine Kurve ist, — dass dagegen im R_n ($n \geq 2$) die Kurven nirgends dicht sind und im R_n ($n \geq 3$) ein zusammenhängendes Komplement besitzen. Im R_2 sind die Kurven mit den nirgends dichten Kontinua (also mit den Kurven im Cantorschen Sinn) identisch. Die Vereinigung abzählbar vieler Kurven ist eindimensional, also wenn sie ein Kontinuum ist, eine Kurve. *

2. *Die Ordnung der Kurvenpunkte.* Jene Kurvenpunkte, zu welchen beliebig kleine Umgebungen mit endlichen Begrenzungen existieren, nennen wir *regulär*. Wenn zu einem regulären Punkt p eine natürliche Zahl $n \geq 1$ existiert derart, dass es beliebig kleine Umgebungen von p gibt, deren Begrenzungen die Mächtigkeit n haben, und n die kleinste Zahl dieser Eigenschaft ist, dann sagen wir, p sei ein Kurvenpunkt der *Ordnung* n ²⁾. Die regulären Punkte, welche von keiner bestimmten endlichen Ordnung sind, nennen wir von *wachsender* Ordnung oder von der Ordnung w . Die nicht regulären Punkte einer separabeln vollständigen Kurve zerfallen in solche von der Ordnung \aleph_0 , das sind jene Punkte, zu welchen beliebig kleine Umgebungen mit abzählbaren Begren-

¹⁾ Vgl. meine Arbeiten über die Dimension von Punktmengen I Teil, Monatshefte f. Math. u. Phys. 1923, S. 148; II. Teil ebenda 1924, S. 137 und Proc. Ac. Amst. Vol. XXVII, p. 639, 1924, sowie die daselbst zitierten Schriften von L. E. J. BROUWER und P. URYSOHN.

²⁾ Monatshefte 1923, S. 156.

4. *Die nicht regulären Punkte.* Ganz allgemein gilt der Satz, dass die Menge aller Punkte des Raumes, für die alle hinlänglich kleinen Umgebungen eine gewisse Eigenschaft E nicht besitzen, entweder leer oder ein kondensierter, in keinem seiner Punkte nulldimensionaler F_σ ist, der in jedem seiner nicht leeren, offenen Teile Kontinua enthält, — wofür über die Eigenschaft E vorausgesetzt wird, dass sie zugleich mit je zwei Umgebungen auch ihrer Summe, und zugleich mit einer Umgebung U auch jeder Umgebung, deren Begrenzung Teil der Begrenzung von U ist, zukommt. Angewendet auf die nicht regulären Punkte, ergibt dieser Satz (der auch der abstrakte Kern gewisser Sätze über die Struktur der n -dimensionalen Mengen ist), dass in jeder Kurve K die Menge K^∞ der nicht regulären Punkte und auch die Menge K^c der Punkte von kontinuierlicher Ordnung entweder leer ist, oder einen kondensierten, in jedem seiner Punkte eindimensionalen F_σ bildet, der in jedem seiner nicht leeren offenen Teile Kontinua enthält. Während die Punkte der Ordnung w isoliert liegen können, liegen also in einer Kurve, wenn sie einen einzigen nicht regulären Punkt enthält, ganze Kontinua von nicht regulären Punkten. Immerhin kann die Menge K^∞ von sehr einfacher Struktur, z. B. ein einfacher Kurvenbogen sein. Die Menge K^c dagegen ist auch selbst von der Ordnung c zumindest in allen jenen ihrer Punkte, in deren Nachbarschaft ihr Komplement $K - K^c$ für jedes $\varepsilon > 0$ mit einer Nullfolge von Umgebungen überdeckbar ist, deren Durchmesser $< \varepsilon$ und deren Begrenzungen abzählbar sind, — insbesondere also ¹⁾ in allen jenen ihrer Punkte, in deren Nachbarschaft die Menge K^c nicht bloss ein F_σ , sondern auch ein G_δ ist.

5. *Zerlegungssätze.* Wir nennen halbkompakt eine Menge, die Summe abzählbar vieler kompakter Mengen ist. Damit zu jedem Punkt einer kompakten (halbkompakten) abgeschlossenen Menge M beliebig kleine Umgebungen einer gewissen Eigenschaft E existieren, ist notwendig und hinreichend, dass M für jedes $\varepsilon > 0$ mit endlich vielen (mit einer Nullfolge von) Umgebungen $< \varepsilon$ von der Eigenschaft E überdeckt werden kann. Wenn zugleich mit den Umgebungen U_1 und U_2 auch $U_1 + U_2$ und $U_1 - U_1 \cdot \overline{U_2}$, (falls diese Menge nicht leer ist) und zugleich mit U jede Umgebung, deren Begrenzung Teil der Begrenzung von U ist, die Eigenschaft E besitzt und wenn ferner die Summe der Begrenzungen von abzählbar vielen Umgebungen mit der Eigenschaft E diskontinuierlich ist, dann ergibt sich auf Grund der sub 4 angeführten Struktursätze weitergehend: Notwendig und hinreichend dafür, dass zu jedem Punkt der kompakten (halbkompakten) abgeschlossenen Menge M beliebig kleine Umgebungen der Eigenschaft E existieren, ist, dass M für jedes $\varepsilon > 0$ Summe von endlich vielen (einer Nullfolge von) abgeschlossenen Umgebungen der Eigenschaft E ist, deren Durchmesser $< \varepsilon$ sind und die

¹⁾ Vgl. K. Menger, Einige Ueberdeckungssätze der Punktmengenlehre, Wien. Ber. 1924 in Druck.

zu je zweien keinen inneren Punkt gemein haben. Diesem Satz zufolge sind die Kurven, bzw. die Kurven von höchstens abzählbarer Ordnung (d.h. die Kurven ohne Punkte der Ordnung c), bzw. die regulären Kurven (d.h. die Kurven, die bloss reguläre Punkte enthalten), unter den kompakten Kontinua dadurch charakterisiert, dass sie für jedes $\varepsilon > 0$ Summe endlich vieler abgeschlossener Umgebungen $< \varepsilon$ sind, die zu je zweien diskontinuierliche, bzw. abzählbare, bzw. höchstens endliche Durchschnitte haben.

6. *Zusammenhang im kleinen.* Ein Kontinuum C heisst nach HAHN (Wien. Ber. 1914 S. 2434) zusammenhängend im kleinen im Punkt p , wenn für jedes $\varepsilon > 0$ alle Punkte einer hinlänglich kleinen Umgebung von p mit diesem Punkt durch Teilkontinua von C verbindbar sind, deren Durchmesser $< \varepsilon$ sind. Als Anwendung der oben angeführten abstrakten Sätze ergibt sich zunächst, dass die Menge aller Punkte, in denen C nicht zusammenhängend im kleinen ist, entweder leer ist oder einen kondensierten, in keinem seiner Punkte nulldimensionalen F_σ bildet, der in jedem seiner nicht leeren offenen Teile Kontinua enthält. Im kleinen zusammenhängend (schlechthin) heisst ein Kontinuum, das in jedem seiner Punkte im kleinen zusammenhängend ist. Unter den kompakten Kontinua sind nach HAHN die im kleinen zusammenhängenden identisch mit den Jordanschen "Kurven", d. h. mit den eindeutigen stetigen Bildern der Strecke. Wenn wir demnach die Kurven im Sinn unserer Definition, welche zusammenhängend im kleinen sind, charakterisieren, so haben wir damit zugleich unter den Jordanschen "Kurven" gerade jene herausgehoben, welche den Namen Kurven mit Recht tragen¹⁾. Nun hatte SIERPIŃSKI (Fund. Math. I, S. 44) gefunden, dass unter den kompakten Kontinua für die Jordanschen Kurven auch charakteristisch sei, dass sie für jedes $\varepsilon > 0$ Summe von endlich vielen Kontinua $< \varepsilon$ sind. Für die im kleinen zusammenhängenden Kurven (im Sinn unserer Definition) ergibt sich nun eine Verschärfung der sub 5 angeführten Zerlegungssätze, welche durch einige Zusatzworte zur Sierpińskischen Charakterisierung der Jordanschen Kurven unter diesen letzteren die wirklich eindimensionalen aussondert: Notwendig und hinreichend, damit ein kompaktes Kontinuum K eine im kleinen zusammenhängende Kurve ($=$ ein eindimensionales stetiges Bild einer Strecke), bzw. eine im kleinen zusammenhängende Kurve höchstens abzählbarer Ordnung, bzw. eine im kleinen zusammenhängende reguläre Kurve sei, ist, dass K für jedes $\varepsilon > 0$ Summe von endlich vielen Kontinua $< \varepsilon$ ist, die zu je zweien diskontinuierliche, bzw. abzählbare, bzw. höchstens endliche Durchschnitte haben.

7. *Die regulären Kurven.* Da jede Zerlegung eines kompakten Kontinuums in endlich viele abgeschlossene Umgebungen $< \varepsilon$ mit zu je zweien

¹⁾ Speziell im R_2 formulieren wir damit eine Bedingung für die Identität von Jordanschen und Cantorsche Kurven.

höchstens endlichen Durchschnitten auch eine Zerlegung zur Folge hat in endlich viele Kontinua $< \varepsilon$ mit zu je zweien höchstens endlichen Durchschnitten, so sehen wir, dass jede reguläre Kurve zusammenhängend im kleinen ist. Darüber hinaus gilt der Satz, dass jede Kurve in jedem regulären Punkt zusammenhängend im kleinen ist. Aber die Menge der nicht regulären Punkte, in denen eine Kurve K zusammenhängend im kleinen ist, muss nicht leer sein; sie bildet einen $F_{\sigma\varphi}$, der auch Punkte von K^c enthalten kann. Es existieren sogar nicht reguläre Kurven, die in jedem ihrer Punkte zusammenhängend im kleinen sind, und nicht reguläre Kurven, die nebst allen Teilkurven in einem irregulären Punkt im kleinen zusammenhängend sind. Andererseits gibt es reguläre Kurven, die Häufungskontinua, d.h. nirgends dichte Teilkurven enthalten. Doch lässt sich vermuten, dass die regulären Kurven unter den kompakten Kontinua neben den oben angeführten Kennzeichnungen auch durch ihre Eigenschaft charakterisiert sind, nebst allen ihren Teilkontinua zusammenhängend im kleinen zu sein.

8. *Die gewöhnlichen Kurven.* Zunächst ergibt sich, dass die einfachen Kurvenbögen unter den kompakten Kontinua dadurch charakterisiert sind, dass sie die einzigen sind, welche abgesehen von zwei Endpunkten ausschliesslich Punkte zweiter Ordnung oder, wie wir sagen wollen, gewöhnliche Punkte enthalten. Jene Kontinua nun, die abgesehen von höchstens endlich vielen End- und Verzweigungspunkten ausschliesslich gewöhnliche Punkte enthalten, nennen wir gewöhnliche Kurven. Zum Punkt p von der Ordnung n einer gewöhnlichen Kurve K existieren genau n einfache Teilbögen von K , welche bloss den Punkt p gemein haben. Die gewöhnlichen Kurven sind, wie sich daraus leicht ergibt, identisch mit den Summen von endlich vielen einfachen Kurvenbögen, die höchstens die Endpunkte miteinander gemein haben. Ist p von der Ordnung n , dann hat die Begrenzung jeder hinlänglich kleinen zusammenhängenden Umgebung von p die Mächtigkeit n . Sind die ganzen, nicht negativen Zahlen $\alpha_1, \alpha_3, \alpha_4 \dots \alpha_n$ vorgegeben, so ist notwendig und hinreichend für die Existenz einer beschränkten gewöhnlichen Kurve, welche ausser α_k Punkten der Ordnung k ($k = 1, 3, 4 \dots n$) bloss gewöhnliche Punkte enthält, das Bestehen der Relationen:

$$\sum_{k=1}^n k \alpha_k \equiv 0 \quad (2)$$

$$\alpha_1 \equiv \sum_{k=3}^n k \alpha_k - 2 \left(\sum_{k=3}^n \alpha_k - 1 \right).$$

Durch diese Sätze wird der *Anschluss an die kombinatorische Topologie* der eindimensionalen Mannigfaltigkeiten hergestellt.

Es sei zum Abschluss bemerkt, dass wir dieser auf eine Dimensionstheorie gegründeten Kurventheorie eine Lehre von den Flächen, den Körpern und den n -dimensionalen Körpern an die Seiten stellen können.

Anatomy. — “*The meninges in Cyclostomes, Selachians, and Teleosts, compared with those in man.*” By Dr. C. U. ARIËNS KAPPERS.

(Communicated at the meeting of November 29, 1924.).

The meninges in lower vertebrates are very different from those in mammals and man.

Though formerly — misled by superficial resemblances — also in Cyclostomes and Plagiostomes, a dura mater, arachnoidea and pia were supposed to exist, at the present time this supposition is only maintained in the “*Mikroskopische Anatomie der Wirbeltiere (Heft IV)*”, published in 1923, by R. KRAUSE, who however does not seem to have studied this subject very accurately.

Already in 1884 SAGEMEHL (l. c.) pointed out that a real arachnoidea does not occur in fishes, and that the widelymeshed tissue formerly considered as such, really lies between the so-called internal and external (periostal) dural membrane, and STERZI ¹⁾ (l. c. 1900—1901) in his comparative anatomy of the meninges emphasized that in Cyclostomes and Plagiostomes only one undifferentiated meninx is found, which he called *meninx primitiva*, and considered to be the origin of the dura, arachnoidea and pia in higher animals.

My own researches confirm STERZI's opinion regarding Cyclostomes and Selachians. It has however to be emphasized that only the internal dural membrane develops from the *meninx primitiva*, the external or periostal dural membrane originating from the endostal (or endochondral) connective tissue that generally in lower vertebrates lies at a great distance from the *meninx primitiva*, and consequently far from the origin of the internal membrane. In my opinion it is better (c. f. also GEGENBAUR ²⁾, POIRIER and CHARPY ³⁾, TESTUT ⁴⁾, STERZI ⁵⁾ and RAUBER ⁶⁾) not to

¹⁾ STERZI. Ricerche intorno all' anatomica comparata ed all' ontogenesi delle meningi. Atti del reale istituto veneto di scienze, lettere ed arti. Anno accademico 1900—01. Tomo 60. Parte II.

See also: STERZI. Recherches sur l'anatomie comparée et l'ontogénèse des meninges. Archives italiennes de biologie, Tomo 37, 1902.

²⁾ GEGENBAUR. (Lehrbuch der Anatomie des Menschen, 6te Auflage 1896, Bnd. II, Page 441) speaks of the „von der inneren Lamelle gebildete eigentliche Duralsack des Rückenmarkes”.

³⁾ POIRIER et CHARPY. Traité d'anatomie humaine, Tome III, 1ier fasc. Paris, 1901, p. 107.

⁴⁾ TESTUT speaks of the internal membrane as „dure mère proprement dite”. See: Traité d'anatomie humaine, 6ième Edition, Paris 1911, p. 1050—1061.

⁵⁾ STERZI. Intorno alla divisione della dura madre dell' endocranio. Monitore zoologico italiano, Anno XIII, 1902.

⁶⁾ Also RAUBER in his Lehrbuch der Anatomie des Menschen (1e Auflage 1903, Bnd. II,

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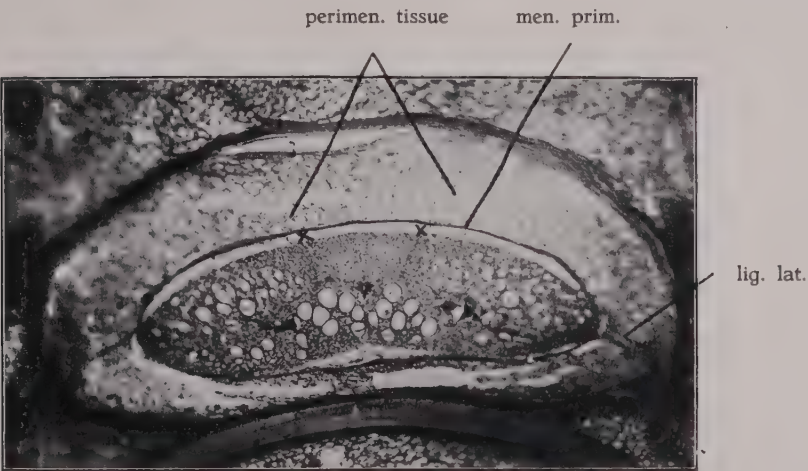


Fig. 1. Spinal cord of Petromyzon in situ
 x x Space between meninx primitiva and cord, caused by retraction.

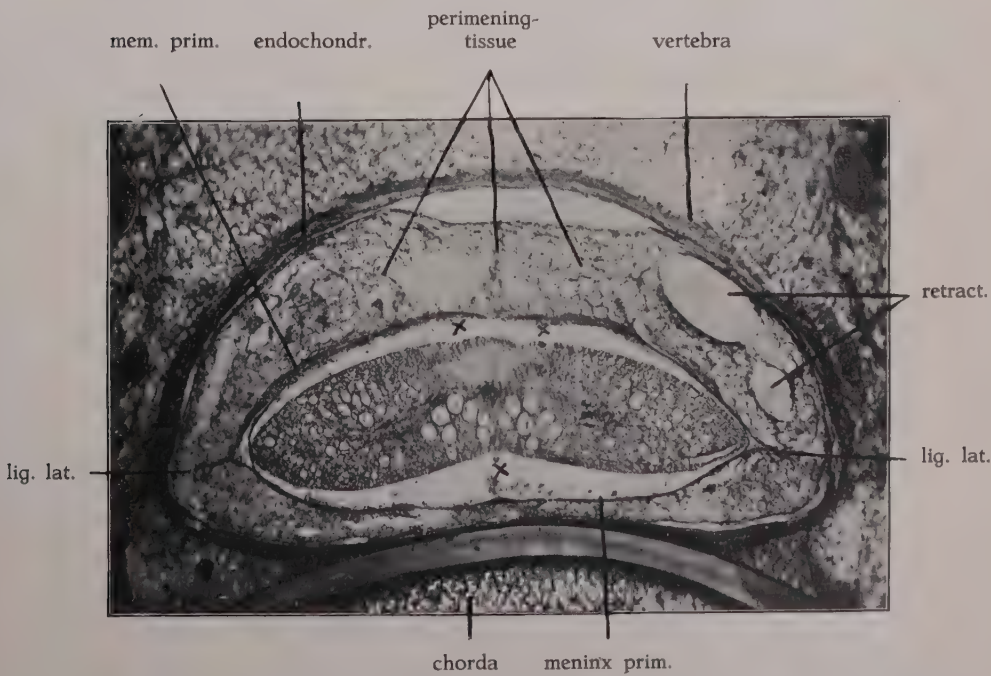


Fig. 2. Spinal cord of Petromyzon in situ.
 x x x Spaces caused by retraction.

consider the so-called external or periosteal membrane of the (spinal) dura (which follows all the sinuosities of the bone) as a part of the dura proper, though it fuses with it in the cranial cavity in the adult (after an embryonic condition in which it may be distinguished from it).

The distinction of a periosteal and an internal layer in the dura mater spinalis only leads to confusion and to the idea of an ambiguous membrane, which ambiguity disappears by leaving the so-called periosteal dural membrane there, where it belongs after its nature and origin, viz. to the connective tissue of the endochondrium or endost and not to the dura.

Concerning the Cyclostomes, I found in *Petromyzon fluvi.* (fig. 1 and 2) relations, as described by STERZI.

The medulla spinalis is surrounded by a single membrane in which no differentiation into separate layers is visible. I call this membrane, with STERZI, *meninx primitiva*. It is continuous with the sheath of the roots.

This membrane, which shows strong lateral ligaments extending far laterally into the perimeningeal tissue (fig. 1, 2) does not yet penetrate with septa into the substance of the spinal cord, so that the membrane easily detaches (XX) from the cord. The nutrition of the spinal cord has to pass everywhere through the superficial glious layer (*limitans superficialis*)¹⁾, there being no intramedullary septa and intramedullary vessels.

Outside this *meninx primitiva*, which, as I shall show later on, also contains the anlage of the dura mater (in the sense of the word mentioned above) lies a broad layer of large cells, which is to be considered as a filling-tissue and reaches as far as the endochondrium of the vertebrae.

This perimeningeal tissue consists of round and oval mucous cells.

In some of my preparations spaces occur, that look like epidural sinusses, which also in human embryos (V. GELDEREN l.c.) are found between the so-called interior layer of the dura and the so-called periosteal layer. I could not find however traces of blood here. They are also more or less local spaces that do not spread over a great length. As I could not find any connection with the *venae invertebrales*, they probably are retraction cavities (caused by the fixation).

The relations in Plagiostomes do not show much difference herewith.

p. 337) speaks of "*Lamina interna oder Dura spinalis in engerem Sinne*". A similar opinion is given in CUNNINGHAM's Textbook of Anatomy (3d Ed. 1909, page 600).

¹⁾ Already in Selachians, meningeal septa with bloodvessels grow into the spinal cord, causing a closer relation between the nervous substance and the vascular system. As however the *limitans gliosa superficialis* grows at the same time with those septa, a real penetration of meningeal tissue into the nervous substance itself does not occur. In fact the septal spaces are to be considered as the fissures in the forebrain, with this difference that they are much smaller and for the greater part filled up with pia tissue, while the arachnoidal cavities in the brain fissures are much wider and go further downwards. They also resemble each other by the fact that the dura remains outside them, in the brain, as well as in the spinal cord.

In *Scyllium* I also found only one meninx, to be called *meninx primitiva*, in which no differentiation in separate layers is visible. It contains small blood vessels, that penetrate with meningeal septa into the spinal cord.

Moreover the sharks show the four spinal ligaments already described by STERZI: the rather strongly pronounced lateral ligaments (fig. 3 and 4), and the thinner, often scarcely developed, dorsal and ventral ligament. Only the lateral ligament extends for some distance through the perimeningeal tissue. The others are merely thickenings of the *meninx primitiva*. Besides this *meninx primitiva* which just as in Cyclostomes, continues in the rootsheath, a large amount of peri-meningeal tissue is found which reaches as far as the endochondrium and shows much more widely spread meshes than in Cyclostomes.

Only here and there — especially in the neighbourhood of the endochondrium — it is a little more compact (fig. 4).

Large thinwalled veins (without muscular coat) are seen in the perimeningeal tissue, especially on the dorsal and lateral side. There is no doubt as far as concerns their homology with the so-called epidural veins in mammals and man, though they are relatively much larger and therefore resemble sinusses.¹⁾

The relations in Ganoids (*Acipenser* and *Polyodon*) are similar to those in Selachians: so I shall not describe them again.

My researches concerning the relations in the most specialized group of fishes, the *Teleosts*, show that there are in this subdivision large differences in meningeal structure that *might* explain the rather different descriptions present in literature, if the chief and rather different descriptions — those of SAGEMEHL and STERZI — were not both based partly on the same material.

STERZI (l.c. I, page 1142) examined especially *Tinca* and further *Cyprinus*, *Esox*, *Barbus*, *Muraena*, *Anguilla*, *Rhombus*, *Solea* and *Labrax*, and found everywhere under the generally perimeningeal tissue only one meninx, which he calls *meninx primitiva*, just as in Cyclostomes and Plagiostomes.

He distinguishes therein two layers, an exterior and interior layer. The *exterior layer* is very thin and consists almost entirely of large flattened cells, more or less pigmented. Further there are star-shaped cells and some cells with long offshoots and connecting ramifications. Between them there are some pigmentcells. In this layer also some elastic fibres are seen. He does not speak of a fibrous thickening of this exterior layer which might indicate a dural development.

The interior layer is more developed. On this²⁾ (in this?) layer, the bloodvessels occur that supply the medulla. It consists of connective tissue fibrils and elastic fibres, crossing each other in different directions.

He further mentions the ligaments and remarks that the sheath of the meninx continues in the rootsheath.

Concerning the perimeningeal space, he remarks that the latter is well developed dorsally

¹⁾ Similar large epidural veins occur in Carnivora, Edentates, Cetacea and Elephas, where the epidural space still prevails on the arachnoidea.

²⁾ The author (l.c. page 1143) says: "Su di esso poggiano i vasi."

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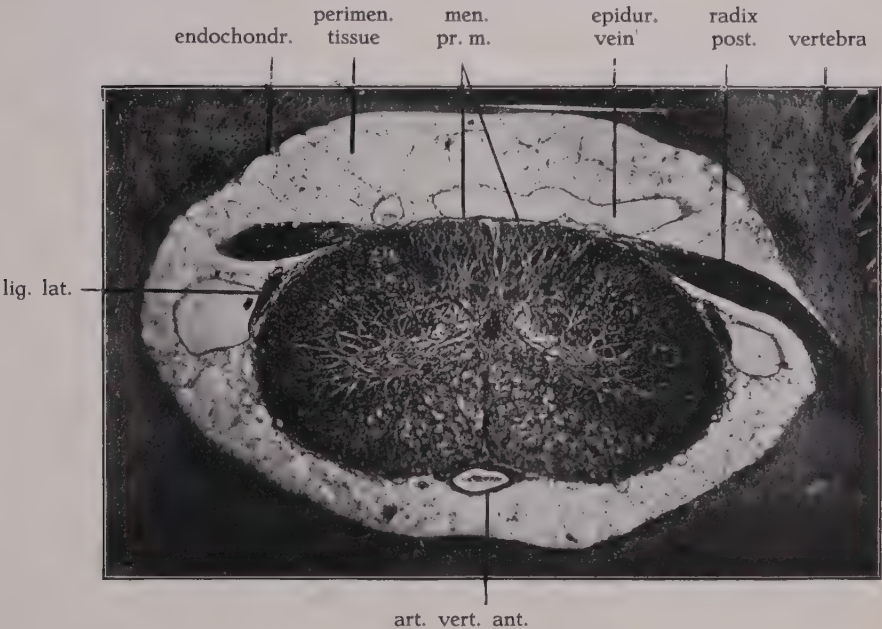


Fig. 3. Spinal cord of *Scyllium canicula* in situ.

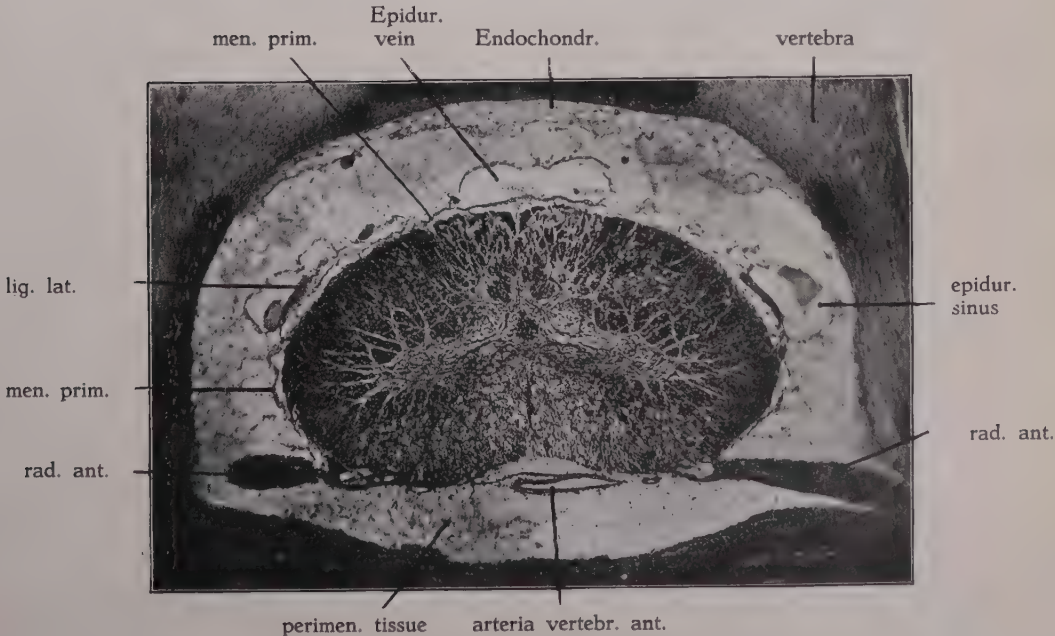


Fig. 4. Spinal cord of *Scyllium canicula* in situ.

and ventrally but may be reduced laterally to a narrow fissure. The perimeningeal tissue¹⁾ in this cavity consists of fine trabecles between which lie large fatcells. Also many small vessels are found there. It has the function of a perimeningeal filling-tissue. Similar relations were found in the other bony-fishes which he examined.

Quite different was the description given before him (1884) by SAGEMEHL²⁾ who in *Siluroides* and *Cyprinoides* (especially in *Barbus* and *Perca*) distinguishes two meninges, distinctly separated by a fissure, which he calls "pericerebraler Lymphraum" and which he considers to be the homologue of the subdural fissure in mammals.

The membrane lying interiorly to this fissure is, according to him, the origin of both the pia and arachnoidea³⁾, although one cannot yet distinguish therein those two membranes. The only distinction to be made in this "vascular-membrane" ("Gefäßhaut" as he calls it) is that only the interior layer of this membrane continues into the fissure mediana anterior, while the exterior layer lies over it like a bridge. Thus far his description of the part lying under the subdural fissure.

Everything outside the "pericerebraler Lymphraum"⁴⁾ SAGEMEHL considers as a dura mater in the old sense of the word, considering as such not only our fibrous dural membrane, but also our perimeningeal tissue and our periostal membrane (l.c. page 460—464).

Resuming we may say that there is a good deal of difference between the opinions of SAGEMEHL and STERZI. STERZI presumes that in fishes there is only one meninx which he calls meninx primitiva, which includes both the dura and the lepto-meninges in an undifferentiated state, while SAGEMEHL distinguishes a dural membrane and under it, separated by a fissure, a tissue which is the origin of pia and arachnoidea (a meninx secundaria as STERZI calls it in Reptiles and Birds). This meninx secundaria ("Gefäßhaut" of SAGEMEHL) shows only in some places a differentiation in an inner and outer membrane, which however has nothing in common with the differentiation in arachnoidea and pia.

The difference between these authors is the more striking as both, at least partly, examined the same material (*Barbus*).

Personal researches convinced me that the relations in Teleosts may be very different. I examined a very small Teleost, *Girardinus*, and compared it with a fish which may attain a very considerable size, *Lophius piscatorius*, and found very different relations.

In *Girardinus* no differentiation is visible in the meningeal tissue sur-

¹⁾ He (and also SAGEMEHL) remarks that the perimeningeal tissue is mucous in Elasmobranchs and Ganoids, and adipose in Teleosts (l.c. page 1147). This is not always correct according to my experience. An *Acipenser sturio* in my collection has for instance a large quantity of perimeningeal fat tissue, and I found mucous tissue in several Teleosts. It seems that both these tissues are most fit to serve as an buffer substance in a movable enclosure.

²⁾ SAGEMEHL. Beiträge zur vergleichenden Anatomie der Fische II. Einige Bemerkungen über die Hirnhäute der Knochenfische. Morphologisches Jahrbuch Bnd. IX. page 457, 1884.

³⁾ For the Teleosts KRAUSE's description is practically in accordance with SAGEMEHL. He, however, does not mention SAGEMEHL's name and considers the interior layer entirely as a pia mater, (l.c. page 647). That the interior layer contains the origin of both pia and arachnoidea, is not mentioned by KRAUSE.

⁴⁾ This word has nothing to do with the so-called epicerebral space of HIS, which this author abusively supposed to exist between membrana limitans gliae and the intima pia.

rounding the spinal cord (and the brain). So here with STERZI one may really speak of one meninx primitiva which also joins the periost, at least laterally where hardly any perimeningeal space is seen between vertebra and meninx (fig. 5).

Dorsally, where the space between the meninx and periost is wider, there occurs in this space a very thin, exceptionally widely meshed perimeningeal tissue, in which, especially at its dorsal side, large veins appear. The same is found in the area of the oblongata and cranium, with this difference however, that there is a much larger quantity of perimeningeal tissue in the much larger cranial cavity. But neither here a differentiation is visible in the meninx primitiva. Consequently, with regard to this animal, STERZI's description is correct.

In *Lophius piscatorius* quite other relations were found. Here also a large quantity of widely meshed peri-meningeal tissue. The actual meningeal tissue lying under it however shows a vere distinct differentiation, in two membranes (fig. 7 and 8).

The outer part of the tissue lying under the widely meshed perimeningeal mucous tissue forms a dense fibrous layer which in some places is larger than in others, but which may be seen everywhere as a distinct layer. If this layer were separated from the underlying meningeal tissue by a continuous split, one would be right in speaking of a well differentiated dura mater. Such a *continuous* split as described by SAGEMEHL and called "pericerebraler Lymphraum" analogous to the subdural cavity in mammals, I cannot find in *Lophius*. The relations here are similar to those observed by VAN GELDEREN¹⁾ in early human embryos. This author found that the (interior layer of the) ectomeninx has become a denser tissue already in human embryos of 19.6 mm. (l.c. I page 2850), contrasting distinctly with the leptomeningeal tissue lying under it without being however separated from it by a fissure, which he even did not yet find in a stage of 25—30 mm., but only saw, occurring first as local dehiscences, in an embryo of 35—40 mm.

The same condition I found in *Lophius*, where I could not perceive a continuous split, but only local dehiscences between the dural membrane and the tissue of the meninx secundaria lying under it. (see fig. 8).

Yet I do not hesitate to consider the fibrous exterior membrane of the meninx primitiva in *Lophius* as dural tissue, as the fibrous condensation proves that it is developing into the direction of the strongly fibrous dura mater and not into the direction of the arachnoidea, which is becoming much more widely meshed. If there were a continuous fissure, then it would not be correct to speak of an exterior dural membrane of the

¹⁾ V. GELDEREN. De ontwikkeling der sinus durae matris bij den mensch.

Ned. Tijdsch. v. Geneeskunde, Vol. 68, 1924, Iste Helft, N^o. 25, Pag. 2850, and Vol. 58 of the Anatom. Anzeiger, 1924, "Zur vergleichenden Anatomie der Sinus durae matris".

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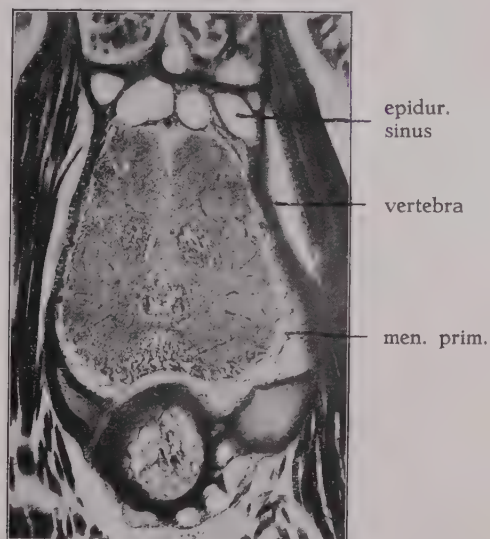


Fig. 5. Spinal cord of *Girardinus* (cervical).
In situ.

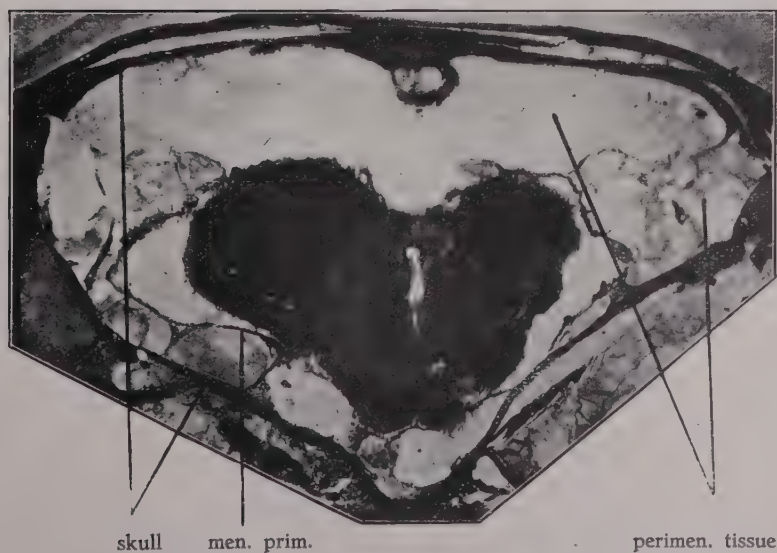


Fig. 6. Frontal part of the midbrain of *Girardinus* in the skull.

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Fig. 7. Cervical cord of *Lophius* in the meninges.

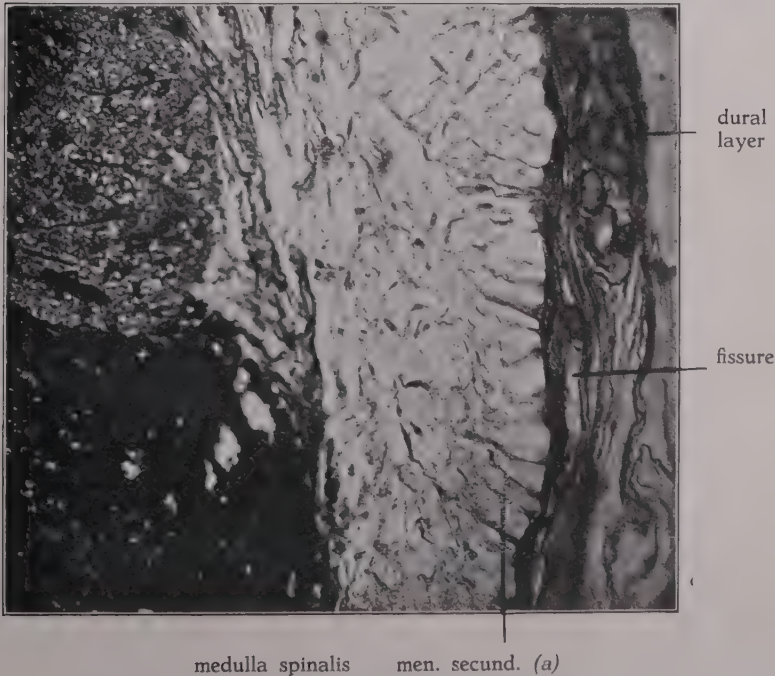


Fig. 8. Enlarged photograph of the meninges in *Lophius*.

meninx primitiva, but of a real dura mater and a meninx secundaria, as they occur in Reptiles.

Concerning the condition of the inner layer of the meninx, its much more widely meshed character is striking (fig. 8—10). In many places we may distinguish in it an exterior layer *a* from an interior layer *b* (see fig. 7—9). In the former, lying directly underneath the mesothelial layer by which it is covered, the cells often stand almost perpendicularly (like palisades) on the external layer of flat cells (fig. 8 : *a*), while the meshes of the interior part are much less regular. Another difference is that only the interior layer follows the fissures and the septa and moreover it contains more small bloodvessels, running in that part that lies immediately on the limitans gliae.

Though there is here a fairly widely meshed tissue, especially in the palisade-part, according to my opinion we may not compare this with the trabecular tissue of the arachnoidea since real "trabecles", that is to say fibrillar threads of connective tissue covered with mesothelial cells, do not occur here. The pseudo-trabecles are ramifications of single cells and consequently might be called *monocellular trabecles* similarly as those occurring in the widely meshed reticular tissue of lymph glands. Moreover in real arachnoid tissue the meshes are much wider and the trabecles far less numerous.

Another argument pleads for this. In mammals the arachnoidea has very wide spaces just at the dorsal side of the oblongata on the choroid of the fourth ventricle (cisterna posterior cerebelli) and communicating with the ventricle by means of the foramen Magendi (where this occurs). In *Lophius*, however, the widely meshed leptomeningeal tissue surrounding the whole surface area of the spinal cord (fig. 7) on all sides, in the area of the calamus (fig. 9) is dorsally a little less developed, and on the choroid roof still less so, being especially developed at the lateral and ventral sides of the oblongata¹⁾ (see fig. 10).

This seems to be in favor of my opinion that this tissue does not yet perform an important function as a receptaculum of ventricular fluid, running into it in higher animals and forming most of the liquor cerebrospinalis externus, but here performs chiefly the same function as widely meshed reticular connective tissue does in other places, f. i. in the intestina and lymph glands.

That this differentiation occurred in *Lophius* and not in *Girardinus* may perhaps be partly due to the much larger space of the vertebral canal in the latter. In larger fishes the skull and vertebral canal increase much more than the nervous system itself, and owing to that the tissue lying between them also increases considerably. That this increase which is very obvious in the perimeningeal tissue in *Lophius*, does not only

¹⁾ Ontogenetically WEED (Anat. Rec. Vol. 10, 1916, p. 479) found the meningeal differentiation also occurring first in the basal parts.

concern the perimeningeal tissue (as happens in the cranial cavity of *Girardinus*, which also is much larger than the vertebral canal of this animal, fig. 6) but in *Lophius* also holds good for the meningeal tissue itself, points however to a higher differentiation¹⁾ to a stage immediately preceding an arachnoidal development.

Thusfar the results of my microscopic research on the meninges in fishes, where the large quantity of perimeningeal, mucous or adipose tissue stands prominent: its quality of a buffer tissue being of great use to the large flexibility of fishes²⁾. We know that traces of a thin perimeningeal adipose tissue still occur in man, in the space between the actual dural membrane and the endost of the vertebrae, while it disappears in the cranial cavity, which is much less subject to changes in form³⁾.

The comparison of the relations between fishes and man, however, asks for a further explanation as far as concerns the development of the arachnoidal spaces and the liquor cerebro-spinalis externus.

There is no doubt that the lowest vertebrates as Cyclostomes, Plagiostomes and Ganoids do not have actual arachnoidal cavities and consequently no liquor cerebro-spinalis externus which in mammals fills the subarachnoidal cavities and whose total volume in man considerably surpasses the volume of the liquor cerebro-spinalis internus (ventricular fluid).

Together with the want of liquor cerebro-spinalis externus, we see the striking fact that the liquor cerebro-spinalis internus — the ventricular liquor — is not seldom very richly developed in lower fishes.

This relative large volume of liquor cerebro-spinalis internus, is not only proved by the wide ventricles in Plagiostomes (especially sharks) and Cyclostomes, but also by the fact that, where these ventricles are covered at the surface by a choroidal membrane, this membrane generally bulges outward considerably, as is shown e.g. in the fourth ventricle and the roof of the midbrain of *Petromyzon* (fig. 11). Also other primitive fishes — f. i. *Ceratodus* (V. D. HORST) — have similar protruding choroid membranes, and with some fishes (*Lepidosteus* and *Amia*), the choroid roof of the third ventricle (the so-called parencephalon) even evaginates in such a degree to all sides that choroidal sacks are formed filled with liquor internus, extending outside alongside the brainwall, far frontally as well as caudally⁴⁾ (fig. 12).

1) *Girardinus* belongs to the Haplomi, that are considered to stand at a lower level than the group of the Pediculati, to which *Lophius* belongs.

2) The fact that the flexions made by fishes in swimming are chiefly lateral may perhaps explain the preponderance in the size and development of the lateral ligaments.

3) Compare also POIRIER et CHARPY, l.c. page 107.

4) ARIËNS KAPPERS. Untersuchungen über das Gehirn der Knochganoiden, *Amia calva* und *Lepidosteus osseus*. Abhandlungen der Senckenbergischen Naturf. Gesellschaft, Frankfurt a/Main, 1907.

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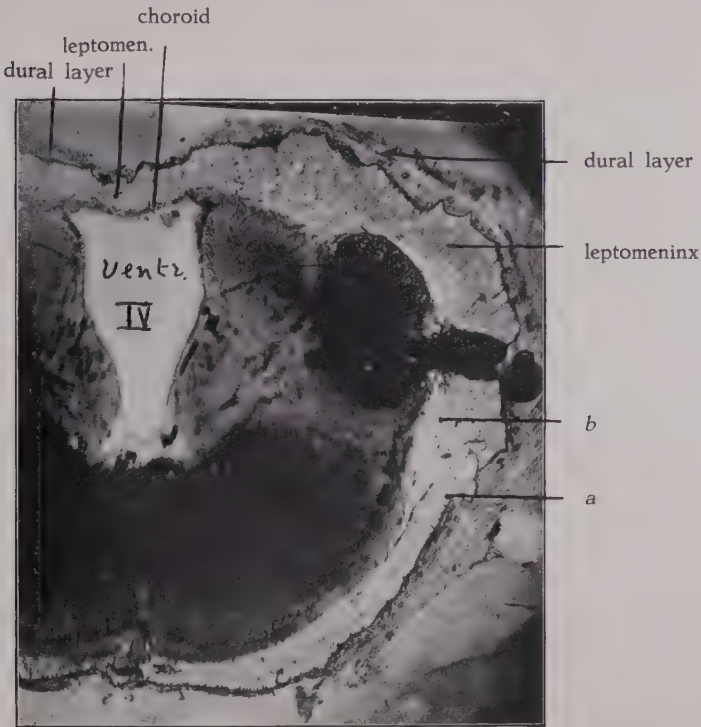


Fig. 9. Lophius. Oblongata on the level of the Calamus.

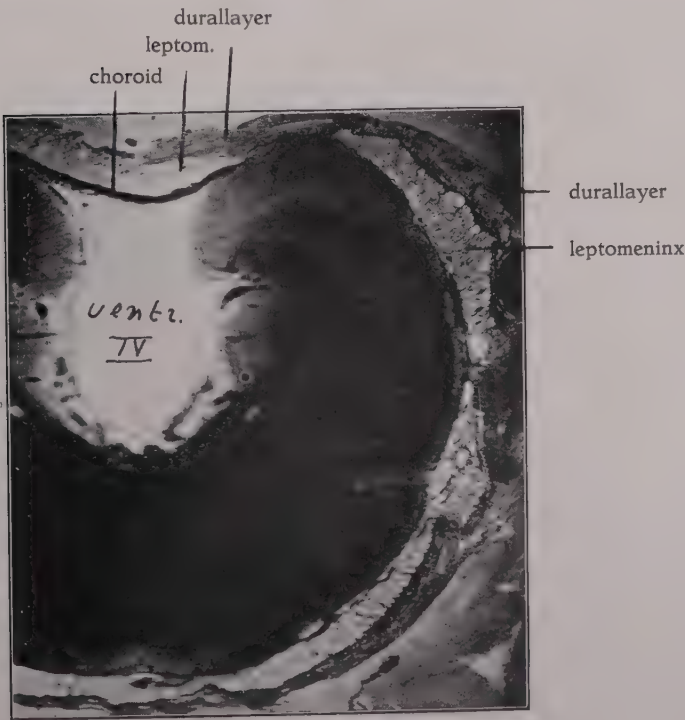


Fig. 10. Meninges of Lophius immediately behind the cerebellum.

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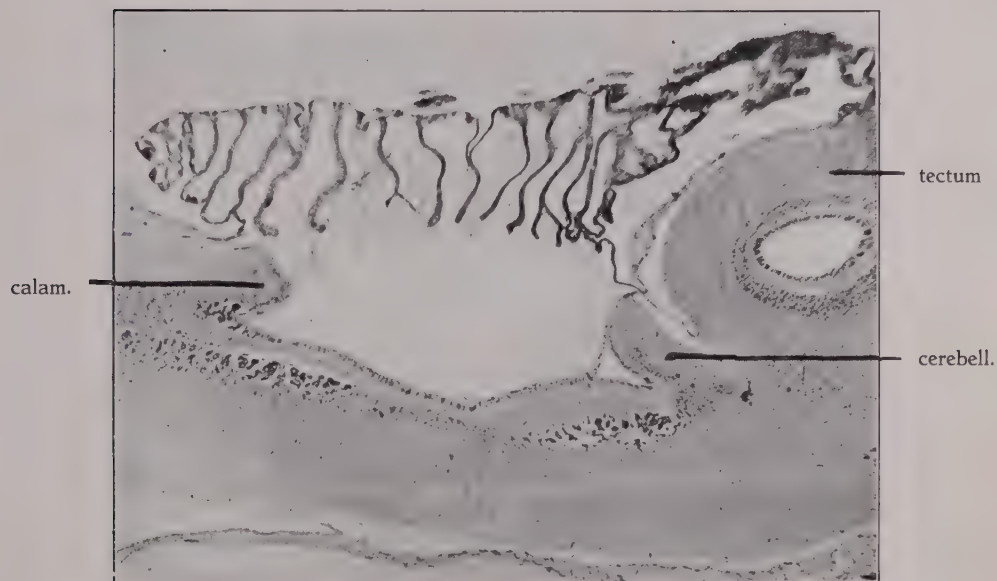


Fig. 11. Fourth ventricle with high choroidal roof in *Petromyzon fluviatilis*; sagittal.

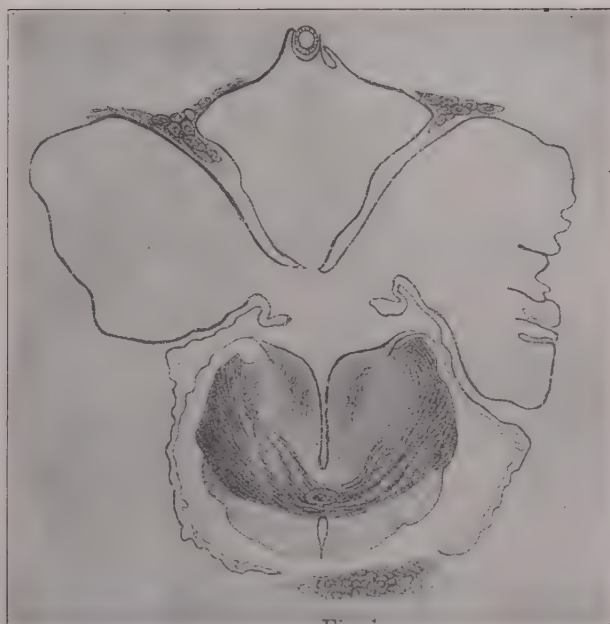


Fig. 12. Cross section through the frontal part of the thalamus with large recessus dorsalis, laterales and ventrales of the choroid of the third ventricle in *Lepidosteus osseus*.

It is evident by all this that the liquor internus has a relatively large volume in many lower animals, in strong contrast to the absence of arachnoidal cavities and liquor externus.

It is interesting that in higher animals, especially mammals — where the arachnoidal cavities with their liquor externus develop very strongly,¹⁾ and at last surpass the volume of the ventricular liquor — the choroid-membranes do no more ude as sacks, but (with few exceptions)²⁾ grow inwards into the ventricles as ventricular draining organs.

In my handbook of comparative brain-anatomy (part II, page 820) I already stated that the coincidence of the accumulation arachnoidal liquor on one hand, and the growing of the choroid membranes into the ventricles on the other (secreting liquor into and at the same time draining the ventricles) is not accidental. It is sure that most of the liquor externus does not originate at the place where it later occurs, but — certainly for the greater part — originates from the ventricular fluid, which diffuses through the choroid-membranes (with or — in most mammals — without assistance of foramina of LUSCHKA or MAGENDI³⁾).

This origin of liquor externus certainly is the most important one, though it may be added that in some places a slight diffusion of ventricular liquor takes place through the ependyma of the ventricles, and arrives into the Virchow-Robin spaces round the brain-vessels and so in the arachnoidal cavities.

In connection with this it is interesting that Dr. FREDERIKSE in the Institute for Brain-research could prove the existence of so-called „Kittsubstanz“ (as also occurs between choroid cells and between intestinal epithelium) between the ependyma cells of the ventricles in the lizard.

In view of the fact that the liquor arachnoidalis originates certainly for the larger part by the diffusion of ventricular fluid through the choroid, it is not strange that the formation of the arachnoidal sacks in mammals arises at the same time with a more draining action and inversion of the choroid.

Of both choroidal functions, viz. the secretion of liquor into the ventricle⁴⁾ at one side, and at the other the draining of ventricular fluid,

¹⁾ Smaller arachnoidal cavities occur already in birds, as HANSEN PRUSS could show by injections. *Journ. of Comp. Neur.* Vol. 36, 1923.

²⁾ The recessus laterales of the oblongata.

³⁾ These foramina are seen for the first time in mammals and do not occur in all mammals. They are even sometimes (but rarely) failing in man.

⁴⁾ The ependyma also takes some part here in, at least in some places (f.i. the infundibulum, see my book, Vol. II, page 821, fig. 437, and page 853, fig. 455 A and B) and the communications of WICLOCKI and PUTNAM. Note on the anatomy of the areae postremae, *Anat. Record* Vol. 19, 1920 and: Further observations on the anatomy and physiology of the areae postremae; *Anat. Record*, Vol. 27, 1924.

the first mentioned function occurs first¹⁾, and this explains as well the strong protrusion of the choroidal sacks in lower fishes, as the absence of proper arachnoidal cavities in these animals.

1) That this process repeats itself in the same order in embryologic development is shown by the researches of LEWIS WEED, who proved that while the ventricular liquid appears in embryos already in the first stage of ventricular development, the liquor externus is found in the arachnoidal cavities of the pig for the first time in an embryo of 14 mM. See his researches: Development of the cerebro-spinal spaces in pig and man: Contributions to embryology, published by the Carnegie Institution, Vol. V, 1917.

Anatomy. — “Some notes upon the finer anatomy of the Brain Stem and Basal Ganglia of *Elephas indicus*”. By Dr. A. PRÉCECHTEL, Prague. (Communicated by C. U. ARIËNS KAPPERS).

(Communicated at the meeting of January 31, 1925).

Until now the brain of the Elephant has been studied chiefly macroscopically, especially by DEXLER ¹⁾, whilst BOLK ²⁾ gave us a minute description of the cerebellum. Concerning its fiber relations, only a description of the rhinencephalon has been given (BINDEWALD ³⁾).

Studying the microscopical sections of *Elephas indicus* in the Central Institute for Brainresearch in Amsterdam, I found several relations in the oblongata, midbrain, thalamus, and basal ganglia, that are of great interest and will be briefly recorded. For comparison several other animals were examined in sections, stained in the same way as the elephants brain, viz. after WEIGERT—PAL en VAN GIESON.

In the cervical cord, the relation of the grey and white matter strikes even the naked eye, the white matter being very abundant, the grey matter relatively scanty. This fact is compatible with the observations made by HOVY ⁴⁾, on a whole series of mammals, proving that concomitantly with the size of the body the mutual relation of the two substances modifies in favour of the white matter.

In the *posterior horn* the *substantia gelatinosa* (fig. I. s. g.) is well developed as it is also in the nearest relatives of the Elephant, the Ungulates. This is still more striking (fig. 2) in the upper cervical region where the strong descending Vth ends.

The *substantia gelatinosa* is arranged in folds, as is frequently the case with a greater development of receptive grey matter in the central nervous system (KAPPERS ⁵⁾) and equally occurs in the spinal cord of Ungulates (BIACH ⁶⁾).

¹⁾ DEXLER. Zur Anatomie des Nervensystems von *Elephas indicus*. Obersteiner's Arbeiten, 1907. Festschrift.

²⁾ L. BOLK. Das Cerebellum der Säugetiere. Bohn, Haarlem, 1906.

³⁾ BINDEWALD. Das Rhinencephalon von *Elephas indicus*. Zoolog. Jahrb. (Anat. und Ontog.) Bnd. 35, 1913.

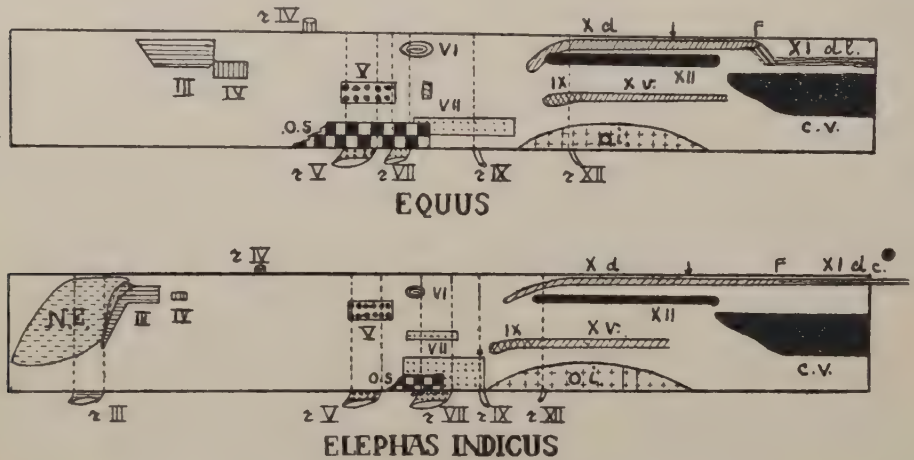
⁴⁾ HOVY. On the relation between the quantity of white and grey substance in the central nervous system. These Proceedings, 16. p. 311.

⁵⁾ ARIËNS KAPPERS. Ueber das Rindenproblem und die Tendenz innerer Hirnteile sich durch Oberflächenvermehrung statt Volumzunahme zu vergrößern. Folia Neurobiologica, Bnd. VIII, No. 4, 1914.

⁶⁾ BIACH Das Rückenmark der Ungulaten. Obersteiners Arb. Bnd. 16, 1908.

The two posterior funiculi distinctly differ from one another by the thickness of their fibres. The neurones of the medial fascicle (GOLL) are thicker than those of the lateral fascicle (BURDACH). This may be explained by the fact that the medial posterior funicular bundle is longer than the lateral posterior bundle, since JOHNSTON¹⁾ has pointed out that an axon often increases in a cellulo-fugal direction.

Dorsally to the *frontal horns* of the cervical region the *nucleus accessorii* is found lying medially in the central zone between the anterior and posterior horns, its root fibres running laterally in a dorsally concave arch (see fig. 2) and leaving the cord between the anterior and posterior root, but nearer the posterior root.



Diagrams showing the difference in topography of the motor nuclei in the horse (VERMEULEN) and Elephant. The arrow indicates the calamus, the flag the hindpole of the dorsal X nucl.

The cells of the nucleus accessorii are polygonal, and form a dorsal column that is not characterized by an invariably equal thickness, but arranged like a string of beads, reaching much further caudal into the cervical medulla than in any other mammal. In cranial direction this nucleus XI passes almost imperceptibly into the column of cells forming the dorsal nucleus vagi. This long caudal extension of the nucleus accessorii in such a dorso-central position is very rare in mammals. Mostly this part of the accessorius nucleus shows a more lateral location, although these lateral cells — as was demonstrated by VERMEULEN²⁾

¹⁾ JOHNSTON. Additional notes on the cranial nervecomponents of *Petromyzon*. Journ. of Comp. Neurology, Vol. 18, 1908. and "The significance of the caliber of the parts of the neurone, ibidem.

²⁾ VERMEULEN. The vagus area in Camelidae. These Proceedings 17, p. 1119. and Die Accessoriusfrage, Psychiatrische en Neurologische Bladen, 1918.

and KAPPERS¹⁾ may show connections with the central or dorsal vagus group. — According to my opinion this dorso-central XI nucleus innervates the *R. internus* XI that joins the vagus peripherally and is sympathetic in function.

I suppose that it is this same nucleus that in Ungulates keeps its attachment with the dorsal X nucleus but for the main shifts more laterally in the intermediary zone. We know from the researches of BOK²⁾ and POLJAK³⁾, that also in the spinal cord a more central and more lateral sympathetic group exists (which are closely related) in the intermediary zone.

I have not been able to trace the homologue of the ventro-lateral XI nucleus innervating the homologues of the sterno-cleido-mastoideus and trapezius in the sections that were available, although its existence is doubtless on account of the muscles and the big size of the spinal XI root.

At the point of transition of the cervical medulla into the medulla oblongata dorsally to the *canalis centralis*, the *commissura infima* appears, containing o. a. decussating fibres of the *fasciculus solitarius* that terminate on the contra-lateral side in the *commissural nucleus* of CAJAL, which is very conspicuous in this animal.

About the same level, the nuclei of the posterior funiculi are seen, first the more caudally situated nucleus of the funiculus of GOLL and more frontally the nucleus of the funiculus of BURDACH, the grey matter of which shows a characteristic lamellar arrangement especially in its caudal part.

The decussation of the spinal pyramids is small, its bundles bending dorsally and running caudally in the *dorsal fascicles* which is the most primitive course of the dorso-lateral pyramids and also found in some Ungulates and Marsupials.

Very striking in the elephant are the strong *fibrae acuatae externae* (f.a.c. fig. 3) running from the nuclei of GOLL, BURDACH and MONAKOW into the region of the inferior olive (*fibrae praetrigeminales*). KAPPERS supposes them to be connected with the inferior olive either directly or by collaterals. He found them strongly developed in Edentates, Anthropoids and man (l. c. p. 214, 215, 598). They run from dorso-caudal into a fronto-ventral direction (unlike the olivo-cerebellar tract).

Of the motor oblongata nuclei the facial nucleus (fig. 5) deserves mentioning on account of the close accumulation of its cells and its distinct division in groups, a

1) ARIËNS KAPPERS. Vergleichende Anatomie des Nervensystems der Wirbeltiere und des Menschen, Teil I, 1920.

2) BOK. Die Entwicklung von Reflexen und Reflexbahnen III. Die Ontogenese des Rückenmarkreflexapparates mit den zentralen Verhältnissen des N. sympathicus. Psych. en Neurol. Bladen 1922.

3) POLJAK. Ueber die Intermediärzone im Rückenmark der Säuger und ihr Verhältnis zum vegetativen Nervensystem. Liječnički Vjesnik (Zagreb, 1924).

phenomenon which probably is connected with the fact that this nucleus innervates the fine stereopractic organ of this animal, the proboscis, whose sensory innervation is largely the trigeminus against whose descending tract the nucleus is located, being bordered on the medial side by the superior olive which may elaborate auditory reflexes on the external and internal ear muscles (M. Stapedius). A dorsal facial nucleus lies above the frontal half of the main nucleus. The VII nucleus receives many pyramidal fibres.

The arrangement of the XII nucleus resembles closely the one described by STUURMAN¹⁾ in the mouse. The nuclei of Roller are very distinct. (fig. 5). The vagus nuclei are well developed, specially the dorsal nucleus, as is generally the case with herbivora, and probably has to be explained by the large stomach of these animals (VERMEULEN l.c.).

Apart from a (rather small) abducens nucleus there seems to be an accessory VI nucl. as described by TERNI²⁾ in rodents, lying near the descending Vth tract.

The motor Vth nucleus is relatively small as usually in herbivora, the largest motor V nucleus occurring in carnivora, whose jaw musculature is much more developed (KAPPERS l.c.).

The system of the Nervus VIII and the cerebellar connections show many primitive peculiarities.

The *nucleus ventralis nervi octavi*, situated ventrolaterally to the corpus restiforme, is not very large and has not shifted as far ventrally as f. i. in Chiroptera and in Carnivora, where the cochlear root is relatively much thicker.

Dorso-medially from the corpus restiforme there even is a cluster of oval cells of the same shape and size as the nucleus ventralis, and which may form a part of the ventral nucleus that — as in *Didelphys* (STOKES³⁾) — has remained in its original position.

Also the *tuberculum acusticum* is little developed and the same may be said of the *oliva superior* (fig. 6, o. s.) nearly entirely situated on the level of the nucleus nervi facialis, medially to it.

Frontally the olive can be traced a little farther than the frontal pole of the nervus facialis, but it gradually disappears in the reticular substance. The relatively small dimensions of the oliva superior are striking but the situation of the facial nucleus so near to it suggests that its reflectory relation with the facial muscles (of the ear) is very close.

The *system n. vestibularis*: The chief vestibular nucleus, the *nucleus of Deiters* with its characteristic large polygonal cells is very strong in the elephant and caudally passes into a large *nucleus vestibularis descendens* (fig. 4), terminating with the *nucleus of Blumenbach* or *Monakow*, which appears as a latero-anterior segment of the *nucleus of Burdach*.

From the Deitersnucleus and the descending VIII cells one plainly sees the fibres of the tractus Deiterospinalis running ventromedially.

The *nucleus triangularis*, medially to the nucleus of DEITERS, consist-

¹⁾ STUURMAN. Die Lokalisation der Zungenmuskeln im Nucl. hypoglossi. Anat. Anzeiger, Bnd. 48.

²⁾ TERNI. Ricerche sul nervo abducente ed in special modo intorno al significato del suo nucleo accessorio d'origine. Folia Neurobiologica, Vol. XII, 1922.

³⁾ STOKES. The acoustic complex and its relations in the brain of the opossum (*Didelphys virgiana*). American Journ. of Anatomy, Vol. 17, 1912.

ing of small cellular elements, is situated quite dorsally under the base of IV-ventricle. Its area is well marked, particularly in caudal levels (fig. 4 X) where it lies medially near the tractus vestibularis descendens and its nucleus, extending caudally a little farther than the commencement of the frontal pole of the nucl. hypoglossi. Here medially from it the tractus solitarius is seen with its grey substance (nucleus parasolitaris) laterally attaching to it.

The nucleus of BECHTEREW lies in the frontal continuation of Deiters nucleus at its typical place near the angle of the ventricle (fig. 5. nu. Bech.). This nucleus consists of cellgroups from which cellular strands run into the cerebellum in a frontal concave arch.

It is possible to trace a direct continuation of this nucleus as far as the medial cerebellar nucleus (nucleus tecti, fig. 5), from which it is obvious that evolutionally the two structures are associated, and that the grey matter of the oblongata has immigrated into the cerebellum (KAPPERS, VAN HOEVELL). To all appearance vestibular fibres ascend to the cerebellum along this strip of grey matter (as has been also described by VAN GEHUCHTEN). Almost completely — but not entirely — separated from the nucleus medialis or the nucleus tecti is the nucleus dentatus (nu. dent.), that is not connected either with the nucleus of BECHTEREW.

Dorsally the dentate nucleus is split, its upper part being situated more dorsally and medially, the other part being situated more ventrally and somewhat laterally.

Frontally the nucleus tecti (of which we know that it is chiefly connected with the anterior lobe of the cerebellum) extends farther than the n. dentatus.

The afferent *connections of the cerebellum* are extremely large, especially the brachium pontis (fig. 6) and the corpus restiforme (fig. 4). In these tracts there are deposits of grey substance that we may designate as the *nuclei brachii pontis* and *nuclei corporis restiformis*. If we trace the sections of the oblongata from the caudal segments, the corpus restiforme, containing reticularly arranged grey matter, begins to appear in the same sections as the posterior pole of the oliva inferior. Frontally the grey substance of the corpus restiforme increases, extending ventral laterally to the tractus trigemini descendens. Dorsally it extends to the nucleus of the descending VIII (see fig. 4). The most external layer of the corpus restiforme does not contain this grey matter. The more frontally, the more the grey matter increases and at the same time extends in strips as far as the ventral pole of the radix trigemini descendens, and occasionally as far as adjacent olive, viz. in those sections in which the tract from the olive inferior attains its greatest volume. More frontally imbedded grey substance diminishes again in the corpus restiforme until it is completely lost on the level of the frontal pole of the inferior olive. Here only from the nucleus DEITERS, i.e. from the frontal conti-

uation of the nucleus of the descending vestibularis, a short strip of grey matter extends between the trigeminus descendens and corpus restiforme.

In more proximal sections, strips of another grey substance appear in the *brachia pontis*, connecting the region of the nucleus DEITERS and its proximal continuation (the nucleus BECHTEREW) with the nuclei pontis. Since (as has already been stated above) the nucleus BECHTEREW is also contiguous with the strips of grey matter extending to the nucleus medialis cerebelli (n. tecti) it results from this that in the elephant there is a bridge of grey matter extending from the nucleus medialis cerebelli to the nuclei of the pons (fig. 5, 6: nu. br. pon.). The distribution of grey matter in the brachium conjunctivum pontis reaches even the most frontal segments of the pons. This appearance of grey matter testifying to the relation of the vestibular area both with the nuclei pontis in frontal and with the corpus restiforme and even with the oliva inferior in caudal direction, is not a regular phenomenon in adult mammals. It may however be well explained by the ontogenetic development of these structures. In 1907 ESSICK has described in man the so-called *corpus pontobulbare*. This is a strip of fibres accompanied by groups of cells and extending from the dorso-lateral somatosensitive static area in a ventromedial direction between the root of the acusticus and facialis, ventrally and frontally. ESSICK ²⁾, confirming the work of HIS ¹⁾, has demonstrated in human embryos as well as in guinea-pig and rabbit embryos the genetical association of the oliva inferior (and nuclei arcuati) and the nuclei pontis with the static area of the bulb, just behind the cerebellum. The corpus ponto-bulbare of ESSICK indicates the path followed by the descending grey matter of this area to the pons. In our sections it is possible to see the relation between the nucleus DEITERS and BECHTEREW and the nuclei pontis, which may be neuro-biotactically understood by the collaboration of the pontine and vestibular apparatus in cerebellar function²⁾.

In the elephant however it is possible to ascertain also the connection of the area statica with the oliva inferior by the grey matter in the corpus restiforme which is exactly co-extensive with the length of the oliva inferior. Also this (fig. 4) may be explained by the embryological work done by HIS and ESSICK and confirmed by KOOY ³⁾ concerning the origin of many cells of the inferior olive from this area which again may be explained by the functional relation between these structures.

¹⁾ HIS. Ueber die Entwicklung des Riechlappens und des Riechganglions und über die des verlängerten Marks. Verhandl. der anat. Gesellschaft, Berlin, 1889.

²⁾ ESSICK. The corpus ponto-bulbare, a hitherto not described nucleus in the human brain. Journ. of Anat. Vol. 7, 1907.

ESSICK. The development of the nuclei pontis and the nucleus arcuatus in man. Journ. of Anat., Vol. 13, 1912.

³⁾ KOOY. The oliva inferior. Folia Neurobiologica, Bnd. 10, 1916. In this paper also an accurate description of the oliva inferior in the adult elephant is found.

In man a small island of grey matter is found constantly in the corpus réstiforme, as ascertained by HIROSE¹⁾ in 1920 (in 13 investigated cases). This nucleus in man is however very small, its long axis measuring 0,5—0,3 mm. in length. Its deposition is constant.

As the nucleus is situated on a level where the oliva inferior is most developed, its relation to the above mentioned structures is probable.

In no adult animal these genetical alliances between the area statica on one hand and the pons and inferior olive on the other are so clear as in the elephant.

The pons grey is very conspicuous. It is distinctly to be seen that the fibrae frontopontinae, situated in the pes pedunculi medially to the pyramid, bend down into the pons before the fibrae temperopontinae. They seem to end sooner²⁾. The fibrae temperopontinae extend a little further caudally before immersing into the pontine cellgroups which, by a strip of grey matter, are connected with the substantia nigra of the midbrain (fig. 7). On the lateral periphery of the isthmus region, two fascicles are striking: the spino-tectal tract (tr. sp. t.) and dorsally to it the ventral spinocerebellar tract (tr. sp. c. v.). More dorso-medially the large *brachia conjunctiva* are seen (comp. fig. 7: br. conj.).

The midbrain.

The *nucleus ruber*, is rather primitive in structure, the majority of its cells being very large magnocellular elements as far the frontal pole. The extension of the nucleus frontally to the emergence of the oculomotor nerve in the elephant is analogous to its relation in Ungulates (goat), as described by v. MONAKOW³⁾.

Laterally to it part of the fibres of the brachia conjunctiva extend farther on and can be frontally traced as far as the thalamus, although it is impossible to determine accurately to which nucleus they run. HORSLEY and CLARKE⁴⁾ assume that this fascicle terminates at least partly in the lateral segment of the thalamus, which may be true also here.

The *substantia nigra* of the pes pedunculi is particularly developed in

¹⁾ HIROSE. A nucleus in the human restiform body. Journ. of the Okayama Medical Association. 1920.

²⁾ This seems to be in favor of the opinion of MASUDA and BOROWIECKI that the fronto-pontine fibres end in cellgroups situated in front of the cellgroups in which the tempero-pontine tract ends.

SPITZER and KARPLUS hold the opposite opinion (c.f. KAPPERS, l.c. sec. p. 733).

³⁾ v. MONAKOW. Der rote Kern, die Haube und die Regio hypothalamica bei einigen Säugetieren und beim Menschen. Arbeiten aus dem neurologischen Institut der Universität Zürich, 1909.

⁴⁾ HORSLEY and CLARKE. On the intrinsic fibres of the cerebellum, its nuclei and its efferent tracts. Brain, 1905.

HORSLEY and CLARKE. Structure and functions of the cerebellum examined by a new method. Brain, 1908.

the median angle of the pes pedunculi (the part called by WINKLER and POTTER¹⁾ nucl. propius pedunculi). Caudally it is continuous with the grey matter of the pons²⁾ which is an evidence of their related function. Frontally it may be followed as far as the globus pallidus, thus proving its relationship to the so called ento- and juxta-peduncular nuclei of reptilia and amphibia. The pes pedunculi in the elephant is also very large in vertical direction and even forces the geniculatum mediale considerably upwards. So it appears that the extrapyramidal motor system is very large in this animal.

Of the *nucl. oculomotorius* in the most caudal sections only the dorso-lateral group is seen, as usually. A little more frontally accedes the ventromedial group.

Frontally the two groups of the nucleus fuse and the nucleus acquires a round form in more frontal sections. About the centre — a little more caudally — appears a new group of cells less intensively coloured, and extending as far as the frontal pole of the nucl. oculomotorii. From this topography we must conclude that this group represents the nucleus *Edinger-Westphal*, serving, according to BROUWER³⁾ a.o., for the innervation of the intraocular muscles.

At its frontal pole the nucleus inclines ventrally and above it begins to appear the large *nucleus ellipticus* which I shall describe in detail further on, together with the hypothalamus with which it is connected (see below). A real central *nucleus Perlia* is not present (it also fails in Ungulates).

The nucleus trochlearis is more primitive than in the horse, lying dorsally, on some distance from the nucleus oculomotorius. It is small and sends its root fibres in the typical way around the aquaeductus Sylvii. Ventrally and close to this nucleus runs the fasciculus longitudinalis posterior. Laterally to this nucleus, a small group of cells is found. This may be a reticular nucleus, as often occurs in this region.

In the mid-brain roof on either side of the aquaeductus Sylvii there are rows of large vesicular cells (fig. 8) that are so large that they can be seen with a moderate loupe. They are the cells of the nucleus of the *mesencephalic trigeminus* from which sensory fibers of the masticatory muscles originate (WILLEMS)⁴⁾. The frontal boundary of these cells coincides with the transverse level on which the frontal boundary of the

¹⁾ WINKLER and POTTER. An anatomical guide to experimental researches on the cats brain (plate XV). Versluys, Amsterdam, 1914.

²⁾ An analogous condition seems to exist in the rabbit if we look at Plate XXI of WINKLER and POTTER'S Anatomical guide to experimental researches on the rabbits brain. Versluys, Amsterdam, 1911.

³⁾ BROUWER. Klinisch-anatomische Untersuchungen über den Oculomotorius Kern. Zeitschrift f. d. gesamte Neurologie und Psychiatrie, 1917.

⁴⁾ WILLEMS. Localisation motrice et kinesthésique. Les noyaux masticateur et mesencéphalique du trijumeau chez le lapin. Névraze, 1911.

oculomotor root is seen and caudally they can be traced even beyond the level of the nucleus nerv. trochlearis. Their considerable size may be explained by the size of the body, as has been shown by G. LEVI¹⁾ (for the spinal ganglion cells) and by OBERSTEINER²⁾ (for the Purkyně cells). It is very striking to see each cell nearly always seated in a capillary bifurcation. With some cells it seems that the capillaries enter the cell as has been observed with the intra- and supramedullary ganglion cells in *Lophius piscatorius* by STUDNIČKA³⁾.

Behind the tectum, in the lateral lemniscus, that can be well traced from the corpus trapezoides, exactly on the level of the issuing point of the root of the trochlearis there is a well developed nucleus of large cells, the position of which (as in the rabbit) corresponds with the ganglion isthmi (fig. 7 g.i.) of Reptilia. Besides there is a well developed geniculatum mediale (fig. 9 g.m.) which may contain the chief part of the g. isthmi of lower vertebrates, the above named nucleus⁴⁾ being the only part of this ganglion that has kept the original place on the level of the trochlear root, the part included in the ganglion geniculatum mediale having shifted in a frontal direction.

Diencephalon.

The *geniculatum mediale* commences as a lateral prominence (see fig. 10), a little frontally to the posterior pole of the nucleus ruber. This structure is strongly developed in the elephant, although the cochlear system (see above) is rather small, thus giving a new proof that this ganglion has still other functions than cochlear, viz. being connected with spino-mesencephalic fibres (WALLENBERG⁵⁾). Frontally the geniculatum mediale leaves the periphery extending in the central grey matter medially to the g. geniculatum laterale in the way as described by INGVAR⁶⁾ approaching the ventral thalamic nuclei. Under it extends a horizontal transverse layer of grey matter lying on the pes pedunculi. This grey substance which I shall call the *nucleus transversus infrageniculatus*, probably

1) G. LEVI. I ganglii cerebro-spinali. Supplemento al volume VII dell' Archivio italiano di anatomia e di embriologia, 1908.

2) OBERSTEINER. Bemerkung zur Bedeutung der wechselnden Größe von Nervenzellen. Volume giubilare in onore di L. Bianchi, Catania, 1913.

3) STUDNIČKA. Ueber die pericellulären und endocellulären Blutcapillaren der Riesenzellen von *Lophius piscatorius*. K.K. Gesellschaft der Wissenschaften. Sitzungsberichte der mathematisch-naturwiss. Classe, 1903.

4) This nucleus is commonly called in mammalian brain anatomy nucl. lemnisci lateralis posterioris; c.f. WINKLER and POTTER: An anatomical guide to experimental researches on the cat's brain, plate XXI (Versluys, Amsterdam, 1914) and MARBURG's Atlas des menschlichen Zentralnervensystems (Fig. 32) Deuticke, Wien 1910.

5) See the general discussion in his paper: Beiträge zur Kenntnis des Gehirns der Teleostier und Selachier. Anat. Anzeiger, Bnd. 31, 1907.

6) INGVAR, On thalamic evolution. (page 700). Acta medica Scand., Vol. 59, 1923

representing WINKLER's and POTTER's¹⁾ c division of the g. geniculatum mediale (fig. 9, 10: nu.tr.i.), extends frontally as far as the geniculatum laterale, a dense layer of fibres separating the structures. Here the nucleus transversus turning in frontal direction laterally and upwards produces a sort of coating for the geniculatum laterale, and finally, extending in front of it, borders the tractus opticus. Some optical fibres pass above this nucleus from which numerous fibres emerge in ventromedial direction. Its function is unknown to me, but its position suggests a relation with the commissura transversa of GUDDEN.

As similar cellgroup occurs in lower fishes, frogs and reptilia.

From the ganglion habenulae the poorly myelinated fasciculus retroflexus (fig. 11) may be easily traced in ventrocaudal direction passing, close in front of the ventral pole of the nucleus ruber and a little medially to it, to the ganglion interpedunculare. Most of the fibres of the fasciculus retroflexus are devoid of myelin, but on its periphery myelinated fibres occur, thus forming a sort of myelin cover for the entire fascicle, as has been described by KAPPERS in fishes (l.c. Bnd. I, p. 80).

The *nuclei anterior and medialis thalami* underneath the ganglion habenulae are clearly separated from the *lateral* and *ventral nucleus thalami*, and the nuclei medialis thalami of either side are united by a broad *commissura mollis*, in which a distinct *nucleus reuniens* is visible. Frontally the medial nucleus (that cannot be with accuracy separated from the anterior nucleus) extends a little further than the lateral nucleus. A pulvinar (fig. 10 pul.) is present, but relatively small. Ventrally to these nuclei we see the *corpus subthalamicum* from which fascicles of fibres run laterally to the globus pallidus (palaeostriatum) above the pes pedunculi (ansa peduncularis, in the field of Forel). It is a circumscribed nucleus, especially so at its medial side (less at its lateral side). Other connections with the striatum are such from the substantia nigra and from the nucleus medialis thalami. The latter connection is distinct along its entire course.

The field of FOREL can be easily distinguished, diminishing caudally where it is more and more replaced by fascicles that near the frontal pole of the nucleus ruber run in a dorsal direction, so that on this level they are longitudinally cut in transverse sections, and are seen passing between the nucleus thalami medialis and lateralis which they separate from one another, and in which to all appearances they enter.

The Hypothalamus and its connections with the oculomotor nucleus.

When describing the oculomotor nucleus, I mentioned that its frontal pole bends in ventral direction. This displacement is caused by a large cellgroup lying in front of the nucleus (diagram and fig. 9—12, nu.e.). This group touches the raphe, and is separated from the cellgroup of

¹⁾ WINKLER and POTTER. An anatomical guide to experimental researches on the rabbits brain, Versluys, Amsterdam, 1911 (plate XIX; text page 2).

the other side only by a narrow raphe of grey matter and nerve fibres. Frontally it reaches as far as the thalamus, extending with its frontal pole under the posterior segment of the nucleus medialis thalami. It is ovoid in form, its longitudinal axis running obliquely from behind and above in frontoventral direction. Its transverse form is flattened. Its caudal and central segment come very near the aquaeductus Sylvii and where the nucleus attains its greatest volume, its upper boundary lies close under the aquaeduct of Sylvius, whilst ventrally it reaches as far as the fornix. Ventrolaterally it passes into the central grey matter. In its frontal fourth we may discriminate in it two divisions, a medial and a lateral one that again fuse at the frontal pole. The cells of this nucleus are small and oval.

In front of it and more ventrally the small c. mammillare is found. The fornix is clearly seen, sending fibres to the corpus mammillare without however completely exhausting itself. On the contrary, the greater part of its intensively staining bundles extend beyond the mammillare reaching the level of the cellgroup described above, even extending beyond its frontal pole running ventromedially to it. These fibres finish in the nucleus ellipticus (see fig. 12: f. f. a.).

Ventrally to this nucleus, commencing with its frontal pole along its entire length, there is a decussation of commissural fibres between the nuclei of the two sides (fig. 12, com.). Other fibres running in the medial plane of the nucleus can be traced in caudal division in the raphe to the nucleus oculomotorius and nucleus EDINGER-WESTPHAL (fig. 11, X).

Fibres from the nucleus, entering the oculomotorius roots cannot be seen (only at the posterior pole this is less certain). It is to be questioned whether or not this particular nucleus may be homologized with any nucleus described until now in other mammals.

PERLIA¹⁾ described a nucleus called by him *nucleus medialis anterior* that according to TSUCHIDA²⁾ is formed by a group of cells assembled in a little oval form with a dorso-lateral longitudinal axis. Its cells by their size and structure remind us of the cells of the nucleus of EDINGER and WESTPHAL. Some authors regard this nucleus as a proximal continuation of the medioventral group of the nucleus EDINGER-WESTPHAL. In the majority of examined cases TSUCHIDA was able to isolate this nucleus fairly well, and he and also KAPPERS consider the original opinion of PERLIA to be correct to the effect that we have to deal with a cellgroup from which no rootfibres originate. So it may be that the nucleus ellipticus is chiefly an enlarged nucl. medialis anterior.

The homologue of our nucleus certainly occurs in the Cetacea where it has been described as *nucleus ellipticus* by HATSCHEK and SCHLESINGER³⁾

¹⁾ PERLIA. Die Anatomie des Oculomotorius beim Menschen. Graefe's Archiv f. Ophthalmologie, Bnd. 35, 1889.

²⁾ TSUCHIDA. Ueber die Ursprungskerne der Augenbewegungsnerven und über die mit diesen in Beziehung stehenden Bahnen. Arbeiten a. d. Hirnanat. Institut in Zürich, 1905.

³⁾ HATSCHEK und SCHLESINGER. Der Hirnstamm des Delphins (*Delphinus delphis*). Arbeiten a. d. Neurolog. Institut der Universität Wien, Bnd. IX, 1902. See also ZWEIG (who saw the same nucleus in *Phocaena*). Jahrb. für Psychiatrie und Neurol. Bnd. 41, 1921.

in the Dolphin. I have also closely examined this nucleus in *Phocaena* and compared its size and its relations to those in the elephant. The only difference is that the nucleus in Cetacea is less flattened in medio-lateral diameter. Moreover in the anosmatic *Phocaena* the fornix is not much developed (and moreover poorly stained in my sections). Hence it was impossible to me to ascertain whether or not fibres of it enter the nucleus ellipticus. On the other hand, however, it was to be seen, also in *Phocaena*, that in the raphe, fibres ascend to the nucleus ellipticus and in *Phocaena* the nuclei of either side are connected by a commissure in exactly the same way as in the Elephant.

It is interesting to note that neither in *Phocaena*, nor in the Elephant it is possible to ascertain a nucleus that would correspond to the nucleus medialis anterior, unless it were the nucl.ellipticus.

That the midbrainbase acts a large part in the search for food in correlation with olfaction is also demonstrated by the presence of the (olfactory) interpeduncular ganglion in all animals in the base of the midbrain. Its relation with vision is expressed by the occurrence of the optic nucl. peduncularis transversus¹⁾ in this region in birds (nucl. opticus basalis), rodents and ungulates. Elephas now gives another striking example of this function, although I could not demonstrate (as I first expected²⁾) a nucl. pedunc. transversus in this animal.

Striatum.

I want to add a few remarks on the *neo-striatum* and *palaeo-striatum* in this animal.

It is easy to discriminate these parts of the basal ganglion. Frontally only the neo-striatum is seen, where the nucleus caudatus with its considerably developed frontal pole fuses with the other neostriatal part, the putamen.

Behind this place these segments are separated from each other by the capsula interna but then they fuse once more at the caudal end, where the capsula disappears again. Frontally the neo-striatum passes into the nucleus accumbens septi as described by E. DE VRIES.³⁾

Medially to the nucleus caudatus on its ventricular side there is a layer of fibres that form a lining of the third ventricle, analogous to the fasc. fronto-occipitalis, but more complete, forming a thin *capsula intima* (fig. 13, X).

Ventrally to these ventricular fibres the taenia semicircularis runs, lying

¹⁾ The homology between the nucl. tr. peduncularis transversus of rodents and ungulates with the ganglion opticum basale of birds has been established by KOSAKA and HIRAIWA (Zur Anatomie der Sehnervenbahnen und ihren Zentren, Folia Neurobiologica, Bnd. 9, 1915).

²⁾ The nucl. tr. peduncularis transversus in which a basal optic bundle ends, is situated between the ganglion mamillare and the substantia nigra closely to the ganglion interpedunculare in Rodents and Ungulates.

³⁾ E. DE VRIES. Das Corpus striatum der Säugetiere. Anat. Anzeiger. Bnd. 37, 1910.

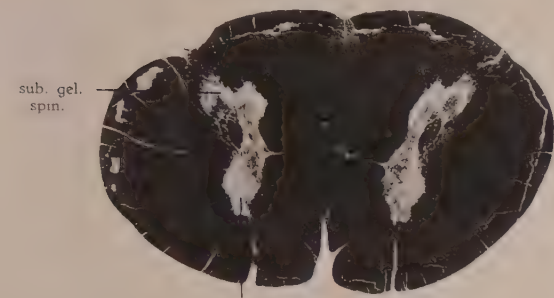


Fig. 1. Cross section of the cervical cord.

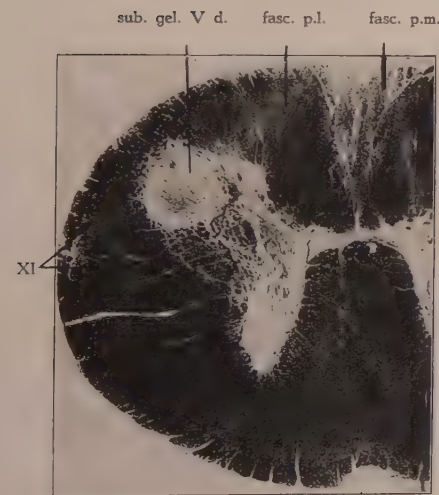


Fig. 2. Cross section of the upper part of the cervical cord.

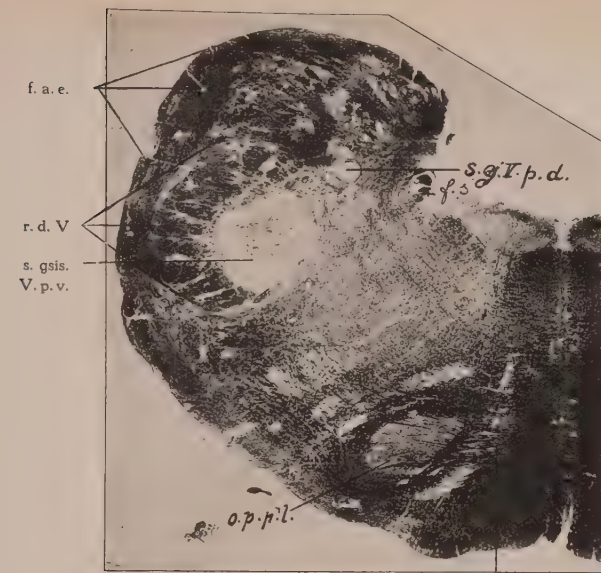


Fig. 3. Section showing the strong development of the fibr. arcuatae externae (f. a. e.) from the nucl. cuneatus externus (Monakow).

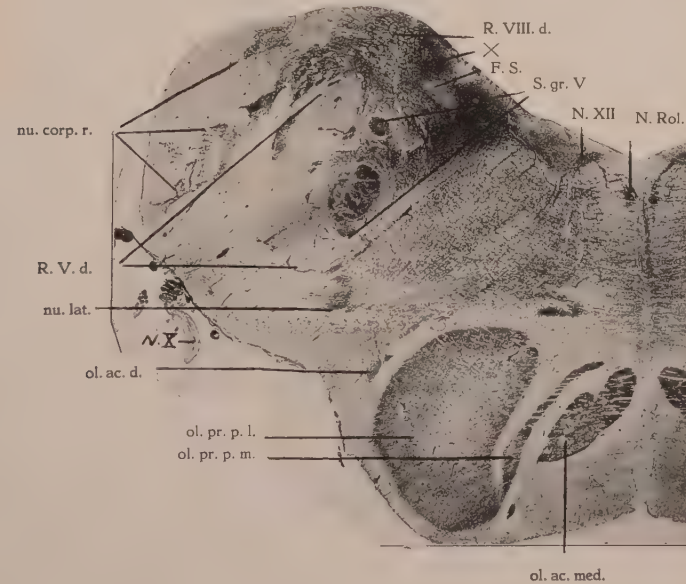


Fig. 4. Section showing the nuclei corporis restiformis (nu. corp. r) and inferior olive.

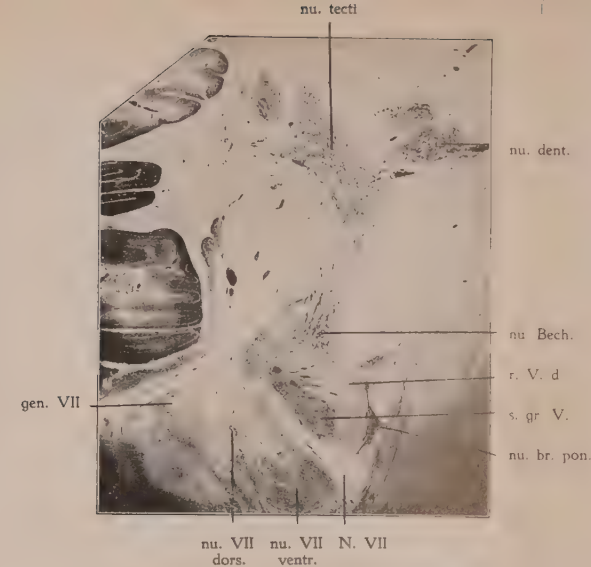


Fig. 5. Nucleus tecti cerebelli and its connection with the nucl. Bechterew. The nucl. brachii pontis and the VII nuclei.

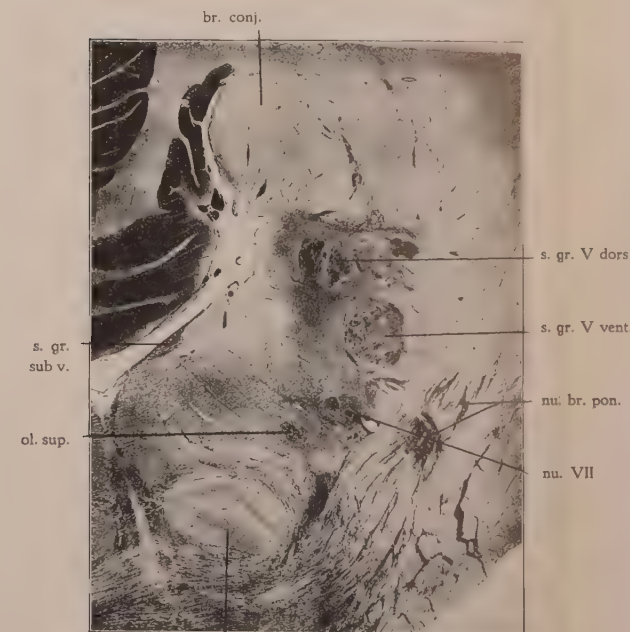


Fig. 6. Showing the nucl. brachii pontis.

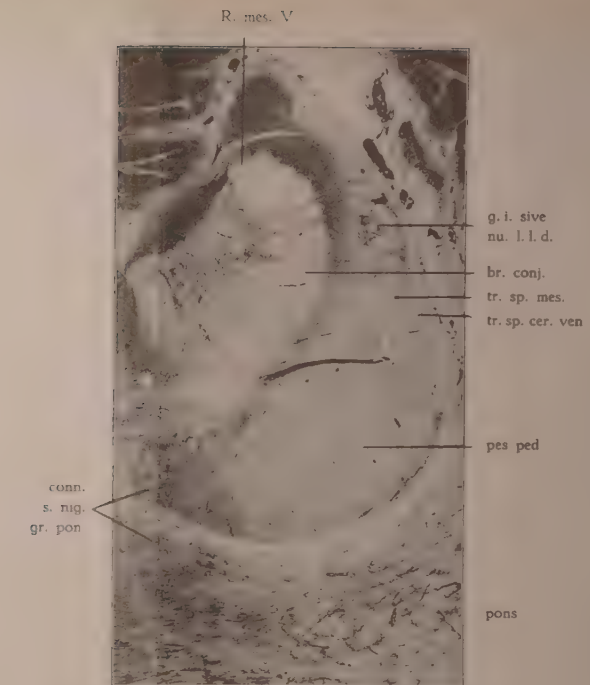


Fig. 7. Showing the connection between the peduncular and pontine nuclei.



Fig. 8. Nucl. mesencephalicus trigemini. The vascularisation of the cells in seen at the bottom.



Fig. 9. The geniculatum mediale with nucl. transversus infrageniculatus.
nu. e. = nucl. ellipticus.

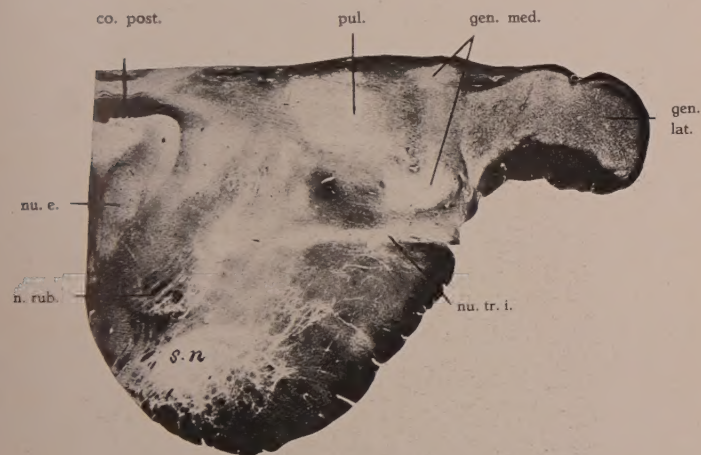


Fig. 10. The nucl. ellipticus (nu. e.); g. geniculata and nucl. transversus infrageniculatus.

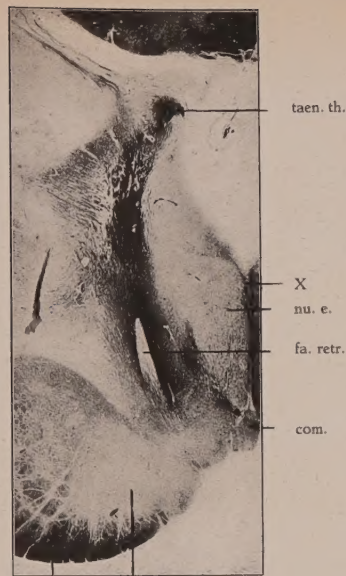


Fig. 11. Showing the commissural (com.) and ascending fibres (X) of the nucleus ellipticus (nu. e.).

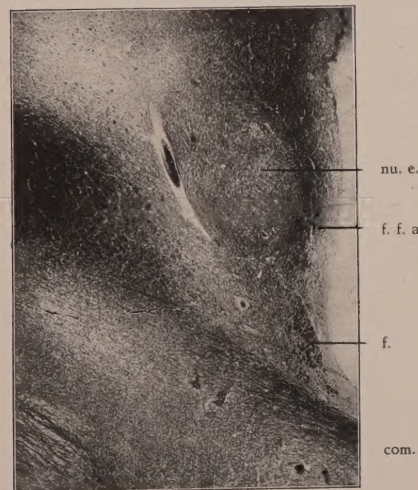


Fig. 12. The fornix (f.) ending with ascending fibers (f. f. a.) in the nucleus ellipticus (nu. e.).

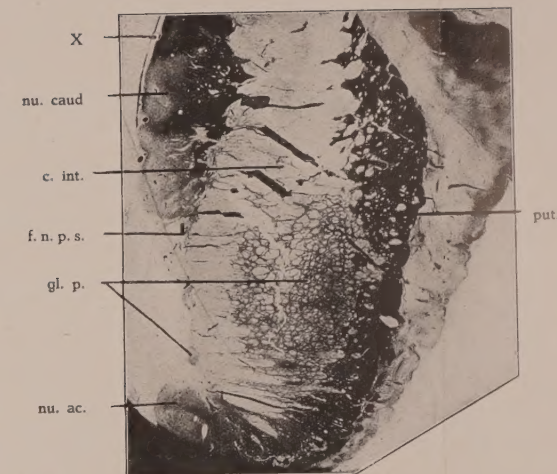


Fig. 13. Showing the subventricular fibers (X) lining the caudate nucleus.
f. n. p. s. = fiss. neo-palaeostriatica. nu. ac. = nucl. accumbens.



Fig. 14. Cauda nucl. caudati (nu. caud.); fissura neo-palaeostriatica (f. n. p. s.) and substantia palaeostriatica caudata. For. = fornix.

ventromedially against the palaeo-striatum. Originating from the regio parolfactoria and running medially to the striatum the taenia semicircularis can be followed as far as the archistriatum or amygdala as in all other mammals, forming the stria terminalis (fig. 14).

In the elephant, more distinct than in most mammals we find medio-ventrally to the nucleus caudatus (fig. 13) KAPPERS' *fissura neo-palaeo-striatica*,¹⁾ which clearly runs backward, separating the cauda of the nucleus caudatus from the substantia caudata palaeostratica (fig. 14), the small strip of grey matter extending backward from the globus pallidus or palaeo-striatum.

The relations of this fissure are similar to those, described in other animals and man by KAPPERS, and like there a bloodvessel is found constantly near this fissure. Moreover a medial extension of the lamina medullaris externa — the borderlayer between palaeo- and neo-striatum — is seen on several places near this fissure, also in the Elephant.

The *palaeo-striatum* or *globus pallidus* at its strongest development has the appearance of reticular substance, extending through the capsula interna with small strips to the ventricle. Its relations to the substantia nigra and corpus subthalamicum have been mentioned above. It also takes part in the formation of the nucleus accumbens (fig. 13: nu. ac.).

The claustrum, as described by ERNST DE VRIES²⁾ is divided into small islands, that are contiguous with the cortex.

¹⁾ ARIËNS KAPPERS. The ontogenetic development of the corpus striatum in birds and a comparison with mammals and man. These Proceedings 26, p. 135, 1922, fig. 17). See also Archives Suisses de Neurologie et de Psychiatrie, Vol. XIII, 1923.

²⁾ ERNST DE VRIES. Bemerkungen zur Ontogenie und vergleichenden Anatomie des Claustrums. Fol. Neurobiologica, Bnd. 4, 1910, p. 496.

